Introduction

A substantial amount of the world's oil and gas production is from reservoirs of friable nature, in which the rate of production is limited by the risk of formation failure in the completion interval. A failure may stop production either permanently or until an expensive workovers can be carried out. In order to minimize the risk of failure, these formations can be strengthened by gravel packing or through one of several available chemical treatments. However, such strengthening measures are expensive and may restrict production or complicate future recompletions to alternative intervals.

Knowledge of completion strength is essential for selecting the optimum type of completion and evaluating the need for a strengthening treatment. The completion decision will be based largely on the maximum allowable sand-free production rate for each candidate completion interval. The completion strength depends on the inherent strength of the formation rock and the stresses imposed on the rock during production. The stresses near the completion depend on many factors including flow rate, well pressure, reservoir pressure and the original in-situ state of stress.

Literature Review

Formation sands having no strength of cementation between sand grains can be stabilized by arches of sand that form around each perforation opening. Some theoretical works has been published on the stability of these arches while producing fluids from the formation.

Bratli and Risnes developed a model by stress analysis on a spherical cavity. They put forward a stability criterion relating the flow forces and the strength parameter in the plastic zone. Their assumptions of spherical cavity is due to the trap door configuration of their experiment where gravity forces tend to make the arch spherical.

Nordgren similar put forward a failure criterion based on stress analysis in hemispherical cavity. His criterion of failure at yield point is too conservative since it had been shown that sandstone surpassed yield point without failure. But his assumption of hemispherical cavity is consistent with laboratory's observation based on downhole configuration.

Stein and Hilchie and later Stein, Odeh and Jones developed method based on the projection of safe and fail production rates from a test well to another wells located in a given area by assuming drawdown or pressure gradient that cause instability in any well is proportional to the sand strength or shear modulus.

For formation with some strength of cementation, tunnels are formed when initially perforated. Some theoretical works has been published on the stability of these tunnels under confining pressure but no works dealing with stability of the tunnels under formation fluid production is available.

The present works is intended to provide a basic theoretical understanding of the mechanical stability of a perforation tunnel in a friable formation during fluid production from the formation.

Model Description

The geometry of the model is as illustrated in Figure 1. It shows a horizontal perforation tunnel through a porous and permeable rock connected to a borehole with the inside radius, \( R_1 \), the midpoint distance between two adjacent perforation tunnels, \( R_0 \) and the perforation length, \( L \). Axial symmetry around the tunnel is assumed and also the rock is assumed isotropic and homogeneous with the pores completely filled with fluid. A plain strain deformation condition is assumed.

Condition of plasticity around a perforation tunnel

The production oil, gas and/or water from an underground reservoir results in a local change in the stress and strain field in a porous, permeable material surrounding the tunnel due to change in pore pressure. For friable sandstone formation the elastic limit can be surpassed without the actual failure.

When critical stresses are reached, the material will be transformed from an elastic to a plastic state. At the transition point, this relation will be the Mohr-Coulomb criterion. But with increasing deformation, the strength of the material may be reduced further, and the constants C and N may be a function of the
Figure 1 Configuration of Perforation Tunnel Problem
degree of deformation and possibly dependent upon the previous history of the material. Also, in the real case there will be a gradual transition from the elastic to the plastic state.

However, for simplicity consider an ideal material which behaves elastically up to the limit given by the Mohr-Coulomb criterion. The parameters C and N may have different values from the ones used in the elastic zone but they are assumed constant for a given material.

**Stress around a perforation tunnel plastic state**

This distribution of stresses around the perforation tunnel is governed by the force equilibrium equation in the plastic zone.

\[ \frac{\delta \sigma_r + \sigma_r}{r} = 0 \]  

... (1)

At the plastic-elastic interface, the relationship between the principal stresses governed by the Mohr-Coulomb yield criterion equation.

\[ f = \delta \theta - \delta_r N^2 + (N^2 - 1) p - 2CN = 0 \]  

... (2)

Assuming a constant flow of fluid into the perforation tunnel, the fluid pressure, \( p \) can be expressed by Darcy’s law as

\[ p = \frac{\mu Q}{2\pi L K_a} \frac{R_n}{r} + P_1 \]  

... (3)

The flow rule associated with (2) is

\[ \varepsilon_r^p = K \frac{\delta f}{\delta \sigma_r} = - KN^2, \varepsilon_\theta^p = K \frac{\delta f}{\delta \sigma_\theta} = K, \varepsilon_z^p = K \frac{\delta f}{\delta \sigma_z} = 0 \]  

... (4)

where, \( \varepsilon_r, \varepsilon_\theta, \varepsilon_z \) is the plastic strain components and \( K \) is a scalar.

From (4) it follows that

\[ \varepsilon_r + \varepsilon_\theta N^2 = 0 \]  

... (5)

And total strain component is defined as

\[ \varepsilon_r = \varepsilon_r^e + \varepsilon_r^p, \varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p, \varepsilon_z = \varepsilon_z^e + \varepsilon_z^p \]  

... (6)

The relationship relating stress and elastic strain is given by:

\[ E \varepsilon_\theta = \sigma_r - \nu(\sigma_\theta + \sigma_z) - (1 - 2\nu) \beta p \]  

\[ E \varepsilon_\theta = \sigma_\theta - \nu(\sigma_r + \sigma_z) - (1 - 2\nu) \beta p \]  

\[ 0 = \sigma_z - \nu(\sigma_z + \sigma_\theta) - (1 - 2\nu) \beta p \]  

... (7)

By combining (2) with (5), (6) and (7) and substituting the resulting expression with the strain-displacement equation into (1) lead to the displacement equation

\[ 2Gu = A_1 r N^2 + A_2 r - N^2 + A' r + A'' r 1nr \]  

... (8)

where: \( A_1 \) and \( A_2 \) are integration constants and

\[ A' = \frac{2(1 - 2\nu)}{N^4 - 1} \frac{1}{N^2 + 1} \frac{(1 - 2\nu)}{N^2 - 1} - \nu \frac{\mu Q}{2 \pi L K_a} \]
The stress distribution around a perforation tunnel can be found by solving (8) for the strain relationships and inserting the resulting expression into (7) with (5), (6) and (3). Combining these solutions with the boundary condition

$$\sigma_r = p_1 \text{ at } r = R_1$$

the final solution for the stress components are given as

$$\sigma_r = p_1 + \frac{\mu Q}{2\pi L K_a} \left[ k_n \frac{R}{R_1} + \frac{1}{t} \left( 2CN - \frac{\mu Q}{2\pi L K_a} \right) \left( \frac{R}{R_1} \right)^t - 1 \right]$$

$$\sigma_\theta = p_1 + \frac{\mu Q}{2\pi L K_a} \left[ 1 + k_n \frac{R}{R_1} + \frac{1}{t} \left( 2CN - \frac{\mu Q}{2\pi L K_a} \right) \left( 1 + \frac{R}{R_1} \right)^t - 1 \right]$$

**Extend of Plasticity around a perforation tunnel**

If failure is expressed in terms of the limiting extension of the plastic zone to the midpoint distance between two adjacent perforation tunnels or any specified distance from the centre of the perforation tunnels as determined by experimental testing of core, then the conditions for failure can be obtained.

At the plastic-elastic interface, the *Mohr-Coulomb* yield criterion hold. Another requirement is continuity in the radial stress across the boundary. When the radial stress from the plastic region and the tangential stress from the elastic zone is inserted into the *Mohr-Coulomb* yield criterion (2), the resulting equation in \( R_C \) is

$$C_1 R_C^{t-2} k_n R_0 / R_C + C_2 R_C^t k_n R_0 / R_C + C_3 R_C^2 + C_4 R_C^2 k_n R_C / R_1 + C_5 R_C^2$$

$$k_n R_0 / R_C + C_6 k_n R_0 / R_C + C_7 k_n R_0 / R_1 + C_8 k_n R_0 / R_C + C_9 k_n R_0 / R_C + C_9 = 0$$

where:

$$C_1 = \left( 2CN - \frac{\mu Q}{2\pi L K_a} \right) R_1 - t$$

$$C_2 = \frac{t + 2}{t} C_1 R_0^2$$

$$C_3 = - \frac{(1 - 2\nu)}{2(1 - 2\nu)} \beta ( P_0 - P_1 )$$

$$C_4 = \frac{(1 - 2\nu)}{2(1 - \nu)} \beta \frac{\mu Q}{2\pi L K_a}$$

$$C_5 = \frac{\mu Q}{2\pi L K_a}$$

$$C_6 = - \frac{(2 - 1 - 2\nu \beta)}{1 - \nu} C_5 R_0^2$$

$$C_7 = - C_4 R_0^2$$
\[ C_8 = (2 \sigma_{RO} - (P_0 - P_1)) \beta \left( \frac{1 - 2v}{1 - v} \right) - 2P_1 + \frac{2}{t} \left( 2CN \right) - \frac{t + 2}{t} \frac{\mu Q}{2\pi LK} R_0^2 \]

\[ C_9 = - C_3 R_0^2 \]

**Result And Discussion**

The example used to illustrate the model developed is given in Table 1.

<table>
<thead>
<tr>
<th>Parameter used for Calculation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pore pressure gradient</td>
<td>0.465 psi/ft</td>
</tr>
<tr>
<td>Formation pressure gradient</td>
<td>0.95 psi/ft</td>
</tr>
<tr>
<td>Perforation diameter</td>
<td>0.5 in</td>
</tr>
<tr>
<td>Perforation shots/ft</td>
<td>4 for 0° phasing</td>
</tr>
<tr>
<td>Formation angle</td>
<td>30°</td>
</tr>
<tr>
<td>Formation strength</td>
<td>217 psi</td>
</tr>
<tr>
<td>Formation compressibility</td>
<td>1.0</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.26</td>
</tr>
<tr>
<td>Flowrate/perforation</td>
<td>10b/d</td>
</tr>
<tr>
<td>Perforation length</td>
<td>12 ins</td>
</tr>
<tr>
<td>Fluid viscosity</td>
<td>0.8 cp</td>
</tr>
<tr>
<td>Formation permeability</td>
<td>100 mD</td>
</tr>
<tr>
<td>Plastic zone permeability</td>
<td>70 mD</td>
</tr>
<tr>
<td>Skin zone permeability</td>
<td>10 mD</td>
</tr>
<tr>
<td>Extent of skin (damage)</td>
<td>0.35 ins from centre of tunnel</td>
</tr>
</tbody>
</table>

**Stress Distribution around a Perforation Tunnel with no Fluid Flow**

The condition of no fluid flow into the perforation tunnel normally assumes that the fluid pressure in the perforation is in balance with the pore pressure in the formation. This would correspond to the situation immediately following the perforating operation.

The distribution of stresses is shown in Figure 2. This thickness of the plastic region in this case extends approximately 0.7 ins from the centre of the tunnel. As long as there is no fluid flow, the thickness of the plastic region depends primarily on the original in-situ stress, pore pressure, the perforation configuration and the formation parameters.

**Stress Distribution around a Perforation Tunnel under Flowing Condition**

This condition corresponds to fluid production from the reservoir into the wellbore through the perforations. The distribution of stresses under flowing conditions is shown in Figure 3. The thickness of the plastic zone increases when compared with the no flow condition assumed in Figure 2. Under flowing conditions, the thickness of the plastic zone is dependent upon the same parameters as the no flow condition and in addition those parameters associated with fluid flow.

**Maximum Sandfree production rate**

Figure 4 shows the maximum sand-free flowrate attainable before perforation tunnel failure occurs. Failure resulted from the expansion of the plastic zone towards the midpoint distance of adjacent perforations or any specified distance from the centre of the tunnel.

**Stability analysis**

The concept of stability ratio is introduced to facilitate the determination of failure. Stability ratio (SR) is defined as the ratio of the radius of the plastic zone to a specified distance from the centre of the tunnel. The effect of the parameters on the SR are plotted for a specified flowrate through the perforation and failure occurs when a specified SR is exceeded. This limit can be determined according to the range of formation encountered and the operational requirements.

The effect upon the SR of both the extent and the properties of the compacted zone around the perforation
Figure 2 Distribution of Stress in Perforation Tunnel
(No. Flow Condition, Elastic-Plastic State)

Examples
Rc = 0.697 ins
Tangential stress
Horizontal stress
Radial stress
Flowrate per perforation = 25 b/d
Re = 0.8636 ins.
Tangential stress
Horizontal stress
Radial stress

Figure 3 Distribution of Stresses in Perforation Tunnel
(Flowing Condition, Elastic-Plastic State)
Figure 4 Maximum sand free flowrate per perforation tunnel

Formation strength
■ 152
+ 112 psi
○ 72 psi
tunnel are shown in Figure 5 and 6, respectively. As the depth of the compacted zone increases the SR increases and higher flowrate through the tunnel tend to increase the SR further. The degree of damage can be seen from the compacted zone permeability and the SR increases with increasing severity of damage.

The effect of perforation parameters as represented by the perforation length, perforation diameter and the number of perforation shots per foot are shown in Figures 7, 8 and 9 respectively. As the perforation diameter increases, the SR increases drastically especially at lower flowrates. A similar trend can be observed with increasing the number of perforations per foot. A shorter perforation length has a higher SR especially at higher flowrates through the perforation.

Figure 10 shows the SR for depleted and geopressed reservoir condition. As the reservoir is depleted the SR increases steeply especially at higher rates of fluid production from the reservoir.

**Conclusion**

This theoretical model of the stresses around a perforation tunnel can be successfully used to study the stability of the tunnel using the plasticity approach and the following conclusions are enumerated below:

(a) The existence of a zone of plasticity is confirmed. Although small, there is a reduction of the stress concentration when compared to elastic consideration.

(b) In no flow condition, the thickness of the plastic region is dependent mainly upon the original in-situ stress, pore pressure, perforation configuration and formation properties.

(c) Limiting the extent of the plastic region to a specified distance from the centre of the perforation is a useful indicator of failure.

(d) Fluid flowing through the formation into the perforation tends to induce instability compared to the no flow case. Damage around the perforation enhanced instability.

(e) The perforation configuration plays a significant role in defining stability.

**Acknowledgement**

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**References**


Figure 5 Stability Ratio vs Extend of Compacted Zone at Various Flowrate

- $Q = 0$ bpd
- $Q = 20$ bpd
- $Q = 50$ bpd
Figure 6 Stability Ratio vs Compacted Zone Permeability at Various Flowrate
Figure 7 Stability Ratio vs Perforation Length at Various Flowrate

Flowrate per perf.
- $Q = 0$ bpd
- $Q = 20$ bpd
- $Q = 50$ bpd

Flowrate per perf., $Q$, values are 0, 20, and 50 bpd.
Flowrate per perf.
Q = 0 bpd
Q = 20 bpd
Q = 50 bpd

Figure 8 Stability Ratio vs Perforation Diameter at Various Flowrate
Figure 9 Stability Ratio vs Number of Perforations/Ft at Various Flowrate

Flowrate per perf.
Q = 0 bpd
Q = 20 bpd
Q = 50 bpd
Flowrate per perf.

$Q = 0 \text{ bpd}$

$Q = 20 \text{ bpd}$

$Q = 50 \text{ bpd}$

Figure 10 Stability Ratio vs Pore Pressure at Various Flowrate
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>inherent shear strength (or cohesive strength), m/Lt²</td>
</tr>
<tr>
<td>cb</td>
<td>rock bulk compressibility, Lt²/m</td>
</tr>
<tr>
<td>cr</td>
<td>rock matrix compressibility, Lt²/m</td>
</tr>
<tr>
<td>f</td>
<td>plastic flow function</td>
</tr>
<tr>
<td>G</td>
<td>shear modulus, m/Lt²</td>
</tr>
<tr>
<td>K</td>
<td>a scalar</td>
</tr>
<tr>
<td>Ka</td>
<td>average permeability, L²</td>
</tr>
<tr>
<td>k</td>
<td>permeability, L²</td>
</tr>
<tr>
<td>kc</td>
<td>permeability in the plastic zone, L²</td>
</tr>
<tr>
<td>ks</td>
<td>permeability of the damage zone, L²</td>
</tr>
<tr>
<td>L</td>
<td>length of the perforation, L</td>
</tr>
<tr>
<td>N</td>
<td>tan²α</td>
</tr>
<tr>
<td>p</td>
<td>fluid pressure, m/Lt²</td>
</tr>
<tr>
<td>pc</td>
<td>fluid pressure at plastic/elastic boundary, m/Lt²</td>
</tr>
<tr>
<td>pi</td>
<td>fluid pressure in the perforation tunnel, m/Lt²</td>
</tr>
<tr>
<td>po</td>
<td>fluid pressure at the midpoint distance between perforation, m/Lt²</td>
</tr>
<tr>
<td>pr</td>
<td>fluid pressure at the edge of the reservoir, m/Lt²</td>
</tr>
<tr>
<td>Q</td>
<td>fluid flowrate, L³/t</td>
</tr>
<tr>
<td>r</td>
<td>radial distance from the centre of the tunnel, L</td>
</tr>
<tr>
<td>R</td>
<td>radial volume force per unit mass, L/t³</td>
</tr>
<tr>
<td>Rc</td>
<td>radius of the plastic zone, L</td>
</tr>
<tr>
<td>ri</td>
<td>radius of the perforation tunnel, L</td>
</tr>
<tr>
<td>Ro</td>
<td>midpoint distance between the perforations, L</td>
</tr>
<tr>
<td>RR</td>
<td>radius of the reservoir, L</td>
</tr>
<tr>
<td>t</td>
<td>tan²α - 1</td>
</tr>
<tr>
<td>u</td>
<td>radial displacement, L</td>
</tr>
<tr>
<td>α</td>
<td>formation failure angle, degrees</td>
</tr>
<tr>
<td>β</td>
<td>1 - cb/cr</td>
</tr>
<tr>
<td>εθ</td>
<td>elastic strain components</td>
</tr>
<tr>
<td>εr</td>
<td>total strain components</td>
</tr>
<tr>
<td>γ</td>
<td>Lame's parameter, m/Lt²</td>
</tr>
<tr>
<td>μ</td>
<td>fluid viscosity, m/Lt</td>
</tr>
<tr>
<td>v</td>
<td>Poison's ratio</td>
</tr>
<tr>
<td>ρ</td>
<td>bulk density, m/Lt³</td>
</tr>
<tr>
<td>σr, σθ, σz</td>
<td>stress components in cylindrical coordinates, m/Lt²</td>
</tr>
<tr>
<td>σR1</td>
<td>stress components at the tunnel wall, m/Lt²</td>
</tr>
</tbody>
</table>
\( \sigma_{RC} \) = stress components at the plastic/elastic boundary, \( m/Lt^2 \)

\( \sigma_{RD} \) = stress components at the midpoint distance between perforations, \( m/Lt^2 \)

\( \phi \) = internal friction angle, degrees.