Synopsis

A gravity basenet has been established in Peninsular Malaysia to meet the requirements of geophysics, geology, metrology and geodesy. The net comprises of 31 stations covering the entire country. The observations of this national gravity base net has been adjusted within the framework of the International Gravity Standardization Net 1971 (IGSN 71). The final adjustment showed a point standard error of better than ±0.05mGal.

The procedures and the results of the adjustment have been reported along with the plans for future work.

Introduction

On realizing the need for a concentrated effort to establish a National Gravity Data Base for the economic and scientific explorations of our land resources, a gravity base network consisting of 31 stations was established during November 1983 through July 1984 covering the entire country as in figure 1. The gravity base net can be used as reference bases for future gravity surveys with an uncertainty of ±0.05mGal or less.
Gravity values of various types are needed for many different purposes: point or mean values, gravity values for anomalies, varying point densities and accuracies. The list given in figure 2 presents an overview of the various applications where gravity values are essential.

<table>
<thead>
<tr>
<th>Application Type</th>
<th>Application</th>
<th>Accuracy Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metrology:</td>
<td>Standard of force</td>
<td>± 1 ... 0.1 mGal absolute</td>
</tr>
<tr>
<td></td>
<td>Mercury-barometer</td>
<td>Topographic reduction</td>
</tr>
<tr>
<td></td>
<td>electric current</td>
<td>Topographic reduction &amp; representation</td>
</tr>
<tr>
<td>Geophysics:</td>
<td>Local studies</td>
<td>± 0.1 ... 1 mGal relative</td>
</tr>
<tr>
<td></td>
<td>Regional studies</td>
<td>± 1 mGal relative</td>
</tr>
<tr>
<td>Geodesy:</td>
<td>Global geoid</td>
<td>± 1 mGal absolute</td>
</tr>
<tr>
<td></td>
<td>Height</td>
<td>± 10 mGal absolute</td>
</tr>
<tr>
<td></td>
<td>Geodynamics</td>
<td>± 3 μGal absolute</td>
</tr>
</tbody>
</table>

Figure 2 Utilization of gravity values.

Net Design, Measurements and Data Processing

Net design

The optimization technique (Wenzel, 1977) was not considered. Instead, I have assumed that the coverage, station spacing and the availability of a fairly good road system play more important roles in the planning of the base net. The International Gravity Standardization Net, 1971 (IGSN 71), (IAG, 1974) stations at Kuala Lumpur (02631 B) and Singapore (02613A) provide the scale and the datum of the base net. The inclusion of the IGSN 71 stations in Penang (02650J) and Malacca (02622J) are not possible, since both sites have been destroyed due to new constructions. The essential data of the IGSN 71 stations are given in Table 1.

Table 1 The essential data of the IGSN 71 stations in Peninsular Malaysia and Singapore.

<table>
<thead>
<tr>
<th>Station Name</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Height (m)</th>
<th>IGSN 71* Gravity Value (mGal)</th>
<th>Std.** Error (mGal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KUALA LUMPUR “B” (02631 B)</td>
<td>3° 06’N</td>
<td>101° 42’E</td>
<td>44.2</td>
<td>978 034.45</td>
<td>± 0.043</td>
</tr>
<tr>
<td>SINGAPORE “A” (02613 A)***</td>
<td>1° 19’N</td>
<td>103° 49’E</td>
<td>19.2</td>
<td>978 066.72</td>
<td>± 0.035</td>
</tr>
</tbody>
</table>

* The Honkasalo's correction term is removed by adding + 0.04 mGal to the IGSN values (Uotila, 1980).

** The standard errors given in the IGSN 71 publication (IAG, 1974) are multiplied by the factor 1.3814775 (Uotila, 1978).

*** The location is now renamed as The Regional Institute of Higher Education and Development, House no. 3. The “bronze disc” has been removed due to some renovations.

1 mGal = 10^-5 ms^-2, 1 μGal = 10^-8 ms^-2
Measurements

In the middle of 1983, reconnaissance trips were made and the proposed base stations were carefully chosen to provide stable, minimum noise, permanent and easy accessible environments. The base stations are documented with photographs and sketches. The horizontal coordinates were scaled from large scale topographical maps while the elevation for some of the stations was determined by levelling from the nearest benchmarks. Few of the stations are located directly at the benchmarks.

Two LaCoste and Romberg model G gravimeters (G-540 and G-542) were used in the measurements of the base net. Model G gravimeters have a range of 7000 mGal and a reading accuracy of ±0.01 mGal. Each measurement consisted of three consecutive readings and was measured to the nearest 0.001 dial graduation in order to eliminate round-off error. A valid observation at a station consists of two consecutive nulls, taken no more than 3 minutes apart, and the reading must repeat to ±0.005 counter units (figure 3 and 4).

![Figure 3](image1)

**Figure 3** A simplified diagram of the LaCoste and Romberg gravimeter.

![Figure 4](image2)

**Figure 4** The two LCR G-540 and G-542 gravimeters at base station no. 30.
Observation sequence of the type ABAB-BCBC— was considered too costly and time consuming. As a solution, longer traverses with overnight stops was then adopted. The measurement was performed so that each gravity difference was measured at least with one forward and one back measurement. The difference in fore-leg and back-leg observations at a station must not exceed ±0.05 mGal (1σ) after earth tide and linear drift corrections are applied (figure 5).

![Drift patterns of G-540 and G-542 gravimeters observed at base station no. 1.](#)

**Reduction of the observations**

**Calibration** — the calibration table as provided by the manufacturer relates the LCR gravimeters counter readings to their relative values in milligals. For example, if the average counter reading observed with LCR G-542 gravimeter is 1625.731, the corresponding value in mGal is $1615.83 + 25.731 \times 1.01059 = 1641.833$ mGal.

**Periodic errors** — the data obtained in the present study are not sufficient for the determination of the periodic errors. Details on periodic errors are clearly described in Krieg (1981). The ideal situation for the determination of periodic errors would be a calibration line which is determined independently from any measuring screw error, e.g., by an instrument with an electronic read out system. Another possibility is to use several different LCR meters in many independent observation series. For this purpose, a calibration line at approximately 10 mGal, one of them being established at 1 mGal intervals has to be realized (see Uotila, 1982). The untreated periodic errors may limit the accuracy of the G-meters to about ± 10 μGal to ± 30 μGal (Harrison et. al., 1978), (table 2).
<table>
<thead>
<tr>
<th>ERROR SOURCE</th>
<th>ESTIMATED ERROR (μGal)</th>
<th>REMARKS ON HIGH PRECISION TECHNIQUES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NORMAL PROCEDURE</td>
<td>HIGH PRECISION PROCEDURE</td>
</tr>
<tr>
<td>INSTRUMENTAL ERRORS</td>
<td>1. reading</td>
<td>&lt; ± 5</td>
</tr>
<tr>
<td></td>
<td>2. leveling</td>
<td>&lt; ± 5</td>
</tr>
<tr>
<td></td>
<td>3. elastic after-effect</td>
<td>&lt; ± 5</td>
</tr>
<tr>
<td></td>
<td>4. instable voltage</td>
<td>&lt; ± 5</td>
</tr>
<tr>
<td></td>
<td>5. calibration terms:</td>
<td>&lt; ± 10</td>
</tr>
<tr>
<td></td>
<td>– linear + quadr.</td>
<td>&lt; ± 10</td>
</tr>
<tr>
<td></td>
<td>– periodic</td>
<td>&lt; ± 10</td>
</tr>
<tr>
<td>ERRORS FROM EXTERNAL SOURCES</td>
<td>1. temperature variation</td>
<td>&lt; ± 10</td>
</tr>
<tr>
<td></td>
<td>2. air pressure variation</td>
<td>&lt; ± 5</td>
</tr>
<tr>
<td></td>
<td>3. magnetic field variation</td>
<td>&lt; ± 1</td>
</tr>
<tr>
<td></td>
<td>4. tides</td>
<td>&lt; ± 10</td>
</tr>
<tr>
<td></td>
<td>5. transportation vibrations</td>
<td>&lt; ± 10</td>
</tr>
</tbody>
</table>
Tidal corrections – the gravitational effect of the sun and the moon on gravity measurements are evaluated using the computer program developed by Heikkinen (1978). Heikkinen's tidal development is based on a completely rigid homogeneous ellipsoid of revolution. The geometric tidal factor of 1.16 is applied to the computed vertical component of the tidal force (Resolution No. 2 of XVII General Assembly of IAG, 1980).

Atmospheric corrections – for high precision gravity measurements it is necessary to correct the observations for air temperature variations and for mass changes in the atmosphere due to air pressure variations (Kiviniemi, 1974). These corrections are essential for places where the daily air temperature and pressure vary rapidly. In Peninsular Malaysia the atmospheric conditions are quite stable. The measurements of the base net was carried out in a free air temperature of about 26° C – 33° C, while the recorded air pressures showed only a small variation of 980 – 1010 mbar. Consequently, for the present study, it is anticipated that the variations in the free air temperature and pressure caused no significance changes on the gravity observations. However, extensive laboratory and field investigations are required to fully understand these effects.

Earth's magnetic field – a fixed orientation with respect to the local magnetic north is observed throughout the measuring campaign in order to minimize the effects of possible beam magnetization.

Clamping error – a waiting time of about five minutes is observed between unclamping and reading the gravimeters in order to minimize the clamping error (also called spring relaxation error).

Drift – drift can occur whenever there is any change in the operating condition of the gravimeter, such as would be caused by vibrations during transportation, meter off-heat, or drastic changes in the atmospheric conditions. The departure of the closing value from the starting value (corrected for tidal effects) of each closed trip was assumed to be mainly due to the instrumental drift, which was distributed linearly with time interval. Strict field procedures (for example, see DMA, 1980) were observed in order to maintain a relatively homogenous drift throughout the measuring campaign.

Adjustment Procedures and Results

Mathematical model for gravimeter

A possible mathematical model for two dial readings at station i and station j can be expressed by:

\[
(y_i - y_j) + \sum_{k=1}^{n} \lambda_k (y_i^k - y_j^k) + (C_i - C_j) - (G_i - G_j) = 0 \quad \ldots (1)
\]

where:

\( y = \) dial readings in mGal
\( \lambda = \) the scale factor term corresponding to the \( k^{th} \) degree, the scale factors is \( \lambda = 1 + \lambda' \)
\( C = \) correction that make the gravimeters observation independent of epoch of the observation
\( G = \) absolute gravity value for the station
\( n = \) the order of the transformation

The expression given by equation (1) is actually an improvement of the formula developed by Uotila (1978) where the problem of scaling the systematic effects has been removed.

A minimum of three absolute measurements (with accuracies of 10–20 µGal) spanning the gravity range of the network is needed to solve for the second and higher order scale factor terms (Uotila, 1978). In the present study, only the linear scale factor term will be solved since the distribution of the two IGSN 71 stations at Kuala Lumpur B and Singapore A is not sufficient for solving higher order terms. Thus, equation (1) is reduced to:

\[
y_i - y_j + \lambda' (y_i - y_j) + (C_i - C_j) - G_i + G_j = 0 \quad \ldots (2)
\]
Mathematical model for the adjustment

The mathematical model of the type $F(L^a, X^o) = 0$ is usually considered as an appropriate model for the adjustment of gravity network (see Uotila, 1974, 1978 and Krieg, 1981). In this case, $X^o$ (adjusted values of parameters) would include the linear scale factor term and the gravity values for each of the base stations. The vector $L^b$ (adjusted values of observables) would have each dial reading of the gravimeter as an element. The usual minimum variance solution for the above model is (Uotila, 1967):

$$\hat{X} = -(A^t M^{-1} A)^{-1} A^t M^{-1} W \quad \ldots (3)$$

where:

$$A = \frac{\partial F}{\partial X^o} \Bigg|_{X^o = X^o} \quad B = \frac{\partial F}{\partial L^a} \Bigg|_{L^a = L^b}$$

$$M = BP^{-1} B^t$$

$L^b$ = observed values of observables

$X^o$ = a priori values of parameters

$\hat{X}^a = X^o + \hat{X}$ -- adjusted values of parameters

The variance-covariance matrix of the parameters has the form:

$$\hat{\Sigma}_{X^o} = \hat{\sigma}^2 (A^t M^{-1} A)^{-1} \quad \ldots (4)$$

where:

$$\hat{\sigma}^2 = V^t P V / (r-u)$$

$V^t P V = -K^t W$

$K = -M^{-1} (AX + W)$

$V = P^{-1} B^t K$ -- the residual vector

$r$ = number of conditions

$u$ = number of parameters

$n$ = number of observations

The weight matrix ($P$) is formed based on the following assumptions:

- The gravity base net for Peninsular Malaysia was carried out according to the "normal procedure" described in table 2, and
- all observations are of equal accuracy and uncorrelated.

Hence, the matrix $P$ can be given as:

$$P = \Sigma^{-1} = \frac{1}{\hat{\sigma}^2} n I_n = \frac{1}{(0.025 \, \text{mGal}^2)} n I_n \quad \ldots (5)$$

Combined adjustment -- this is the case where we have observations made in two groups (LCR G-540 and LCR G-542) and we have the same unknown parameters in both cases. Let $L^1_b$ and $L^2_b$ are the two groups of observables, the mathematical model can be given as:

$$F_1(L^1_b, X^o) = 0 \quad \ldots \quad \text{LCR G-540}$$

$$F_2(L^2_b, X^o) = 0 \quad \ldots \quad \text{LCR G-542}$$
The above system can be related to equation (3) as follows:

\[
B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}, \quad A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}, \quad L^b = \begin{bmatrix} L_1^b \\ L_2^b \end{bmatrix}
\]

\[
W = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \quad P_1 = P_2
\]

\[
\hat{X} = -(A_1^t M_1^{-1} A_1 + A_2^t M_2^{-1} A_2) (A_1^t M_1^{-1} W_1 + A_2^t M_2^{-1} W_2)
\]

\[
\Sigma_{x_a}^\lambda = \sigma (A_1^t M_1^{-1} A_1 + A_2^t M_2^{-1} A_2)^{-1}
\]

\[
\delta_0^2 = V^t P V / (r_1 + r_2 - u)
\]

\[
V^t P V = V_1^t P_1 V_1 + V_2^t P_2 V_2
\]

The IGSN 71 stations at Kuala Lumpur B and Singapore A were held fixed and errorless (static case). The priori values for the parameters were taken from a preliminary adjustment carried out by Radzi (1985). Stations no. 9 and 10 were not included in the adjustment due to poor connections made to these stations. The solutions are iterated until some set of limits have been reached.

Adjustment results

The $V^t P V$ obtained for the LCR G-540 was found to be too large than for LCR G-542. As a consequence, the variance-covariance matrix for the adjusted parameters for the G-540 data-set also found to be significantly larger than expected. The results of the combined solution also suffered from the poor performance of the G-540 gravimeter. Instrumental problems such as meter off-heat, beam vibrations and tares could be the contributing factors to the poorly determined results with the G-540 data set. Gravimeter G-540 may also have been poorly handled and maintained during the field campaign.

Therefore, the final gravity values of the base net is based only on the results of adjustment with the G-542 data set. The following is the summary of results of the adjustment with the G-542 data set:

| Number of observations | 72 |
| Number of parameters   | 30 |
| Number of gravity stations | 29 |
| Number of scale factor term | 1 |
| Number of iterations   | 3 |
| $V^t P V$              | 28.706 |
| Degree of freedom      | 36 |
| a posteriori variance of unit weight | 0.7974 |
| gravimeter scale factor, $1 + \lambda'$ | 0.99939 |

Conclusions and Plans For Future Work

Of the three adjustments carried out, the adjustment using the data set from LaCoste and Romberg G-542 gravimeter has been considered as optimum for the time being. With this adjustment, the gravity values at 29 base stations can be considered to have been established with an accuracy for most of the stations better than 0.05 mGal. The established base stations will facilitate the unification of the regional gravity surveys for geodetic and geophysical applications into a uniform reference system.
For the improvement of the present base net, the following work is proposed:

- Establishing the absolute value of gravity at some selected stations covering the gravity range experienced. The absolute stations can be established in collaboration with the Finnish Geodetic Institute in Helsinki, employing a portable absolute gravimeter (Kakkuri, private communication, 1982).

- Investigations on the calibration and periodic errors of the gravimeters. Special calibration line as proposed by Uotila (1982) needs to be realized.

- Performing adjustment with improved mathematical models such as those with the inclusion of parameters for the drift coefficient and environmental factors.

- Observational schemes used in the present study need to be improved for better drift control. Diligence of the observer is strongly recommended in any future gravity surveys in order to reduce the uncertainties of LCR gravimeter measurements.

Acknowledgments

The groundwork for the gravity base net project was initiated in 1982 during my study leave at the Finnish Geodetic Institute in Helsinki, Finland. I wish to gratefully acknowledge the valuable help given by Professor Juhani Kakkuri, the Director of the Finnish Geodetic Institute. Bureau Gravimetric International in Tolouse, France has kindly provided the essential data and descriptions of the IGSN 71 stations in Peninsular Malaysia and Singapore.

The field measurements was carried out through funds supplied by the Universiti Teknologi Malaysia.

Professor Uotila, my present advisor at the Department of Geodetic Science and Surveying, the Ohio State University, helped clarify many of the adjustment problems and provided valuable advice on numerous occasions.

The computer time used in this study was provided by the Instructional and Research Computer Center (IRCC), the Ohio State University.

Finally, I would like to thank my colleagues at the Universiti Teknologi and the Survey Department of Malaysia for having contributed in some way or the other to the results presented in this paper.

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