Abstract. Holographic measurement of modulus elasticity was performed on samples of aluminum can. It is based upon the measurement of the displacement as a function of number of fringes appeared on the body of the loaded can. The value of the modulus elasticity for the aluminum can is obtained as $(1.00 \pm 0.03) \times 10^8 \text{ Nm}^{-2}$. This result is found in good agreement with a mechanical measurement by taking into account any experimental error.

Keywords: Holographic interferometry, double exposure, displacement, modulus of elasticity, fringes, laser

1.0 INTRODUCTION

Holographic interferometry is an extension of interferometric measurement techniques in which at least one of the waves that interfere is reconstructed by a hologram [1].

This method gives rise to interference patterns whose fringes are determined by the holographic setup via the sensitivity vectors and optical path length differences. Thus holographic interference patterns can be produced either by keeping the optical path length difference constant and changing the sensitivity vectors, holding the sensitivity vectors constant and varying the optical path differences, or altering both parameters together. The path lengths can be modified by a number of physical parameters. The flexibility and precision gained by comparing the optical path length changes with the wavelength of the laser light used makes holographic interferometry an ideal means for measuring a manifold of physical quantities [2,3,4].

In double-exposure holographic interferometry, interference takes place between the wavefronts reconstructed by two holograms of the object recorded on the same photographic plate. Typically, the first exposure is made with the object in its initial, unstressed condition, and the second is made with a stress applied to the object. When the processed hologram is illuminated with the original reference beam, it reconstructs two images, one corresponding to the object in its unstressed state, and the other corresponding to the stressed object. The resulting interference pattern reveals the changes in the shape of the object between the two exposures [1].

Several studies have been reported on the measurement of deformation by holographic interferometry [5,6,7]. In elastomechanics, the interest in measurement of

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the strains and stresses is far superior than the measurement of the displacements of a deformed object [8].

This paper reports the experiment work on double exposure technique to measure the displacement of aluminum can, due to given variable loading. By knowing the displacement, the strain of the aluminum is calculated or determined. Hence, the elastic modulus of the aluminum can could be calculated.

2.0 THEORY OF HOLOGRAPHIC INTERFEROMETRY-
DOUBLE EXPOSURE TECHNIQUE

Holographic interferometry is defined as the interferometric comparison of two or more waves, one of which is holographically reconstructed. Since the object and reference waves come from the same laser source, they have a high degree of mutual coherence and produce an interference pattern on the plate [9]. Essentially the method consists of recording two successive holographic exposures of an object in two different conditions [10].

The mathematical description of double exposure holography can be best derived by first considering that in the scalar diffraction theory, the linearly polarized monochromatic light can be represented by its complex amplitude $A$:

$$A(x, y, z) = a(x, y, z) \exp[-j\varphi(x, y, z)]$$  \hspace{1cm} (1)

where $a(x, y, z)$ is the real amplitude of the light wave, and $\varphi(x, y, z)$ is the phase of the light wave [10].

A holographic plate is exposed at time $t_1$ to wave $A_1$, diffused by the object under test, together with a reference wave $R$ and then, at time $t_2$ to wave $A_2$, diffused by the deformed object together with $R$, where:

$$A_1(x, y, z) = a_1(x, y, z) \exp[-j\varphi(x, y, z)]$$  \hspace{1cm} (2)

$$A_2(x, y, z) = a_2(x, y, z) \exp[-j\varphi(x, y, z) + \Delta \varphi(x, y, z)]$$  \hspace{1cm} (3)

Deformations of the object mainly affect the phase of the wave. When the developed hologram plate is illuminated by the reference wave $R$, the resulting reconstructed wavefront will have a term of the form $(A_1 + A_2)$, which is proportional to the image wave front.

The intensity distribution in the reconstructed image is then:

$$I(x, y, z) = |A_1(x, y, z) + A_2(x, y, z)|^2$$

$$= a^2(x, y, z)2[1 + \cos \Delta \varphi(x, y, z)]$$  \hspace{1cm} (4)

Equation (4) represents the background intensity of the object, $a^2(x, y, z)$, modulated by a system of the fringes, $2 [1 + \cos \Delta \varphi(x, y, z)]$, which is a function of the phase
change resulting from object deformation. Dark fringes are contours of the constant values of $\Delta \varphi$, which are odd-integer multiples of the $p$, while bright fringes are contours of the constant values of the $\Delta \varphi$, which are even-integer multiples of the $\pi$. Due to sensitivity to surface deformations, the technique can be used to gain meaningful information with regards to the structural characteristics of a component, by observing the surface movement produced when the component is subjected to a mild stressing force. As such it offers the potential for inspection problems where the defect of the interest can be made manifest as an anomaly in an otherwise regular interferometric fringe pattern; the stressing technique must be devised in such a way that the anomalies induce detectable perturbations in the surface deformation [10].

During reconstruction, the two images interfered as though there were two copies of the object present. Where the displacement between the two “copies” is such that the reflected light would be shifted by an integral multiple of half wavelengths, there will be a dark fringe in the combined image. Conversely, bright fringes occur when the displacement is an integral number of wavelengths.

The optical geometry is shown in Figure 1(a) and (b). The change in optical path length, $\Delta O$, due to the shift in position $\Delta y$ of the surface is:

$$\Delta O = (z_1 + z_4) - (z_2 + z_3)$$  \hspace{1cm} (5)

Using the given angles this becomes:

$$\Delta O = \Delta y (\cos \alpha + \cos \beta)$$  \hspace{1cm} (6)

where $\alpha$ and $\beta$ are the incident and view angle, respectively. The conditions for interference can be written as:

$$\Delta O = \frac{n\lambda}{2}$$  \hspace{1cm} (7)

where $\lambda$ is the wavelength of the light source and $n$ is the integer number of fringes line from some undeflected reference point. If odd integers imply dark fringes and even integers imply bright fringes, the quality of image should be improved. It will be better if the images are redrawn. Equating these two expressions (6) and (7), the formula to be used in the analysis is:

$$\Delta y = \frac{n\lambda}{2 (\cos \alpha + \cos \beta)}$$  \hspace{1cm} (8)

### 3.0 MODULUS OF ELASTICITY

The modulus of elasticity is a fundamental material constant and is an index of the stiffness of the material. For many common structural materials, the strain is essentially
linear function of the stress over the range of stresses normally used in load carrying members [11, 12].

Young’s Modulus:

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{\Delta y / L}$$

where, $F$ is the loading force in Newtons, $A$ is the contact surface area of the Aluminum can in $m^2$, $\Delta y$ is the displacement, $L$ is the height of the aluminum can, and $E$ is the modulus of elasticity measured in Pascals (Pa).

4.0 METHODOLOGY

The optical alignment of the experimental set up is shown in Figure 2. A 10 mW He-Ne laser 632.8 nm was used as a light source. The cylindrical aluminum can was employed as an object. The dimension of the aluminum can is 132.76 mm in height with a diameter of 52.5 mm. It was sprayed with a light coat of flat-white paint to have a good reflection when illuminated with the laser beam.

The holographic exposure is made of two steps, the first half exposure is made for the normal object without stress and the other half is made after the object is given stress by load on top of the can. The plate was exposed for 10 seconds. This procedure is called a double exposure. The photographic emulsion was manually developed by using developer-19, stopped bath and fixer. Then, the plate was washed and dried.

The hologram was reconstructed by illuminating the same reference beam such as shown in Figure 3. The incident angle of the beam upon the plate is $a = 30^\circ$ and the angle of view is also $30^\circ$. The load on the can was increased with an increment of 100
gram. The interference pattern appeared on the corresponded body was recorded. In this study, the minimum load responsible for the appearance of the interference pattern on the body of the aluminum can was found to be 10 gram and the maximum loading in which the interference pattern was still visible was about 2000 gram.

The images of the hologram are permanently recorded using a Nikon COOLPLX995 digital camera. It is important to fix the camera position. All developed plates were viewed using exactly the same plate holder. This is important to determine the magnification factor and angle of the incident beam.

**Figure 2** Optical set up of interferometry hologram (double exposure)

**Figure 3** The reconstruction set up
5.0 RESULTS AND DISCUSSION

The typical results obtained from this experiment are shown in Figure 4.

The frames are arranged in the order of increasing loading. No fringes appeared after the can was loaded with greater than 2000 gram, such as shown in Figure 4(x). As the load on the aluminum can is increased, the displacement of the aluminum can body also increased. The displacement of the aluminum can keeps on increasing even after the load is more than 2000 gram, as long as the yield point of the aluminum

![Figure 4](image_url)

**Figure 4** Different interference patterns on double exposure holography
can is not exceeded. However, due to the limitation of the coherent length between the first and the second image, in order to maintain superposition among the waves, the interference fringes soon get blur and finally disappear. This phenomenon is manifested in Figure 4(u) to 4(x).

The interference pattern is an indicator for the occurrence of the deformation on the object. The displacement is calculated by counting the number of fringes or lines under the measured area. The fringes lines are counted using Matrox inspector 2.1 software, under 1 cm$^2$-measured area. Two spots, which are about 1 cm apart, are drawn on the exposed body for the calibration of magnification factor.

For each hologram plate, at least 10 random places were chosen for counting the fringes under 1 cm$^2$-measured area. An average fringes number was taken as a number of fringes counted for each loading.

The deformation profile of the aluminum can is obtained by plotting the number of fringes under 1 cm$^2$-measured area, against the correspond loading. The graph is shown in Figure 5.

Initially, the curve is constant even when the load increases, meaning that the number of fringes is maintained as one within that region. The number of the fringes started to rise as the loading was greater than 600 gram. At this stage, the elastic property of the aluminum can is manifested, whereby, the number of fringes was found linearly proportional to the given loading.

![Figure 5](image-url)  
**Figure 5** The profile of deformation on aluminum can
The strain of the aluminum can was calculated by using equation (8). The information required in the equation comprised of the number of fringes, the incident angle, and the view angle, which is 30° respectively. The displacement is then divided by the original height of the can to obtain the strain.

The stress on the aluminum can is calculated by dividing the magnitude of given loading with the surface contact area. The stress is then plotted versus strain, as shown in Figure 6. The average elastic modulus of the aluminum can was obtained as $(1.00 \pm 0.03) \times 10^8 \text{ Nm}^{-2}$. This result agrees with the direct measured value using Universal Testing Machine. The mechanical measurement of Young’s Modulus was obtained as $1.76 \times 10^8 \text{ Nm}^{-2}$. This shows that the experimental result is in a good agreement with the mechanical measurement by taking into account several factors including measurement and calculation error.

### 6.0 CONCLUSION

The deformation of the aluminum can due to the loading effect can be detected using the holographic interferometry double exposure method. The qualitative result can be obtained from the hologram image, which showed the deformation of the object was used to quantify the Modulus elasticity of the aluminum can. The experimental result agreed with the direct measurement using Universal Testing Machine.
ELASTICITY MEASUREMENT USING HOLOGRAPHIC INTERFEROMETRY

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