DIMENSIONAL ANALYSIS AS APPLIED TO SCOURING AROUND BRIDGE PIERS IN ALLUVIAL RIVERS

ABDUL AZIZ BIN IBRAHIM
Fakulti Kejuruteraan Awam

RINKASAN

Percubaan telah dibuat untuk menganalisa masalah hakisan dasar di sekitar tiang jambatan di dalam sungai lanar dengan penggunaan cerakan dimensa.

Proses hakisan sekitar tiang jambatan dipengaruhi oleh sebilangan akubah angkubah kompleks yang berkaitan. Hakisan di sekitar tiang jambatan didapati bergantung kepada dimensi jambatan dan saluran, Nombor Froude, nisbah pengecutan, ciri-ciri bahan dasar, dan parameter-parameter berhubung dengan alat-alat mengawal hakisan.

Data terdahulu juga digunakan bagi menyokong kemasukan dan penyisihan beberapa angkubah yang penting.

SUMMARY

An attempt has been made to analyse the problem of the erosion around bridge piers in alluvial rivers with the help of dimensional analysis.

The scouring process in the vicinity of a bridge pier is controlled by a number of complex interrelated variables. Scour around bridge piers is shown to depend primarily on the dimension of the pier and channel, Froude Number, contraction ratio, properties of the bed material and parameters related to devices controlling the scour.

Previous data are also employed to support the inclusion or exclusion of some of the important variables.

Introduction

Fluid motion is complex phenomenon. It is therefore, not yet possible to define fluid motion completely by analytical methods. To assess the flow pattern around a complex engineering structure, recourse is taken to study the phenomenon using a small scaled model.
The model is designed on the principles of similitude. These are of three kinds, viz., geometric, kinematic and dynamic, thus denoting similarity in form, motion and forces respectively. Conditions of similitude may be determined by applying the principles of dimensional analysis.

In free surface hydraulics, flow depends on the forces of inertia and gravity, properties of the flow and the boundaries, to name only a few. Dimensional analysis is used as a tool to study the significant groupings of these parameters. Among the dimensionless groupings which are of importance are Froude, Reynolds, Weber and Cauchy numbers. These groups should be of the same value both in the model and in the prototype for complete similitude.

Unfortunately, it is not possible to make all significant numbers equal, in a reduced scaled model and in its prototype. Generally, a scaled model does not reproduce all aspects of the phenomenon under study. It models only a few aspects that are of interest in the research work. Therefore, it is necessary to show that the effects of other groupings are negligible.

Basic Parameters

The following analysis is limited to the case of an isolated bridge pier in a channel whose flow is assumed to be steady and uniform. The variables entering the problem may be grouped into the following categories:

(i) Parameters describing the fluid — the fluid is described by its density \( \rho \) and viscosity \( \nu \).

(ii) Bed material, cohesionless granular material — cohesionless granular materials are classified according to grain size \( D_g \) and density \( \rho_s \). The absolute size of the sand is determined by the specification of the absolute value of any single grain size, selected as typical. Therefore, any diameter \( D_g \) (i is any percentage) of a given mixed granular material can be selected as the typical diameter \( D_g \), in order to specify the absolute size of the material.

(iii) Flow — a steady, uniform open channel flow is characterised by its average depth \( h \), velocity \( v \), and by gravity \( g \) which generates the flow, and width \( B \).

(iv) Pier — the characteristic parameters that define the pier are size \( d \), shape and surface condition. Quantitatively, only size \( d \), is considered.

(v) Protection device, the ring-plate — the effective use of the device depends on its position above the bed level and its size relative to that of the pier. Hence, characteristic parameters of the plate are position \( y \), and size \( D \).

It follows that the depth of the scour hole \( d_s \), in uniform flow involving a cohesionless bed material near the vicinity of a bridge pier is defined by the following set of eleven characteristic parameters:

\[
d_s = f(p, \nu, \rho_s, D_g, h, v, y, B, \text{and } D) \quad \cdots \cdots \cdots \cdots \quad (1)
\]

Application of Dimensional Analysis Technique

For practical applications, certain parameters in equation (1) can be substituted by other dependent parameters. For example, either \( P \) or \( P_s \) can be substituted by \( \Delta = (p_s - \rho)/\rho \), the relative submerged density. Dynamic viscosity can be replaced by \( \nu = \mu/p \), the kinematic viscosity. The final variables included in the dimensional analysis are given in Table 1.0. The dimensionless version of a function of \( n \) characteristic parameters, such as those given in Table 1.0 can be determined by the \( \pi \) Theorem of The Theory of Dimensions.

From dimensional reasoning and selecting \( P, h, v \), as basic quantities, the similitude coefficients given in Table 2.0 are obtained. The maximum depth of scour can be expressed by the function:

\[
d_s (\text{max})/h = f(Re, Fr, Dg/h, d/h, y/h, D/h, B/h, \Delta) \quad \cdots \cdots \cdots \cdots \quad (2)
\]

where \( Fr \) is the Froude number \( (U^2/gh) \) and \( Re \) is the pier size Reynolds number \( (gd/v) \).

The influence of the pier size Reynolds number \( (gd/v) \) was clearly shown by Shen et al (1966, 1969) who produced a plot of Reynolds number versus scour depth based on laboratory data. This is reproduced here as Figure 2.0. From Figure 2.0 it can be seen that for any given size of pier and bed material, as \( Re \) increases, scour decreases until a certain limiting value is attained. Therefore, scour depth remains constant.

Essentially each line in Figure 2.0 represents the relationship between scour depth and velocity since pier size is constant and it is unlikely that variations in viscosity were significant. At first sight it would appear improbable that scour depth would decrease with increasing velocity or indeed attain a constant value but this might well be the result of increasing bed-load activity. If so, then the range of diagram is beyond the scope of the present study which is confined solely to the circumstances of clear-water scour i.e. local scour with no general bed movement. From the point of view of the dimensional analysis Figure 2.0 is incomplete in that the effects of flow depth are not indicated. If they were it might be that the apparent anomaly would be clarified. Leaving these points aside, it would seem that the influence of pier size Reynolds number at most is only valid over a limited range of flow conditions and as such can probably be omitted from the dimensional relationship without greatly reducing the accuracy of the final result.

From the foregoing results of findings it can be seen that it is the author's opinion that any relationship relating scour and the characteristics of the water
flow must contain both velocity and depth. These characteristics may be most conveniently incorporated in the form of the Froude number as has been shown in many previous studies such as those of Chabert and Engeldinger (1956), Batz (1960) and Chitale (1962). In all cases it has been found that both field and laboratory data plotted as the ratio of maximum scour depth to flow depth ($d_s$ (max)/$h$) against Froude number may be fitted by a straight line. Also included were data from author of previous study (1982). This is shown in Figure 3.0. It would appear that the author's data follow a slightly different trend from the other data. However it could be argued that the trend line from the author experiments fits the previous data just as well as the Batz's (1960) curve. Thus for a wide range of data and experimental conditions a relationship arises with a fair degree of scatter can be established between scour depth in the form of the ratio scour depth to flow depth and the flow parameters of depth and velocity in the form of the Froude number. In the circumstances of the author's previous experiments (1982) which were conducted at a constant depth of flow the relationship effectively reduces to one of scour depth against velocity of flow. Figure 3.0 on this basis shows that scour depth increases continuously with increasing velocity within the clear-water scour regime. This is entirely logical, showing none of the anomalies of the Reynolds number relationship, and thus confirms the validity of the Froude number as a basis of the local scour rating curve.

The term $\Delta$ is constant if one consider only natural sand (i.e. $\Delta = 1.65$).

The dimensionless groups $y/h$ and $D/h$ are kept constant alternatively in physical model test programmes. Also, the dimensionless group $d/h$ is constant for each test programme.

The final parameters involved are as in equation (3).

$$d_s (\text{max})/h = f_2 (F_i, y/h, D/h)$$

Conclusions

1. The scour process around bridge piers in alluvial rivers is a very complex 3-dimensional flow involving numerous parameters, too many to be dealt with by purely empirical means.

2. An extensive use of dimensional analysis, to some extent, help to reduce the number of parameters sufficiently for experimental verification of the problem.

3. Froudian criterion should be satisfied, viz., the velocity scale must be proportional to the square root of the depth scale ($v = \sqrt{gh}$).

4. If a ring-plate is to be used as a protective measure, its lateral extent $D$, and its vertical position $y$, is critical.

### Table 1.0 Variables used in the Scour Problem

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid density</td>
<td>$P$</td>
<td>$ML^{-3}$</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>$\nu$</td>
<td>$L^2T^{-1}$</td>
</tr>
<tr>
<td>Relative submerged density</td>
<td>$\Delta$</td>
<td>Dimensionless ratio</td>
</tr>
<tr>
<td>Particle diameter</td>
<td>$D_p$</td>
<td>$L$</td>
</tr>
<tr>
<td>Approach flow depth</td>
<td>$h$</td>
<td>$L$</td>
</tr>
<tr>
<td>Mean velocity</td>
<td>$\bar{v}$</td>
<td>$LT^{-1}$</td>
</tr>
<tr>
<td>Gravity</td>
<td>$g$</td>
<td>$LT^{-2}$</td>
</tr>
<tr>
<td>Pier size</td>
<td>$d$</td>
<td>$L$</td>
</tr>
<tr>
<td>Position of plate</td>
<td>$y$</td>
<td>$L$</td>
</tr>
<tr>
<td>Ring plate size</td>
<td>$D$</td>
<td>$L$</td>
</tr>
<tr>
<td>Channel width</td>
<td>$B$</td>
<td>$L$</td>
</tr>
</tbody>
</table>

### Table 2.0 Similitude Coefficients for the Scour Problem Around Bridge Pier

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\frac{v_d}{D}$</td>
<td>Pier Reynolds Number</td>
</tr>
<tr>
<td>2. $\bar{v}$</td>
<td>Froude Number squared</td>
</tr>
<tr>
<td>3. $D_p/h$</td>
<td>Relative bed material size</td>
</tr>
<tr>
<td>4. $d/h$</td>
<td>Relative pier size</td>
</tr>
<tr>
<td>5. $y/h$</td>
<td>Relative position of ring plate</td>
</tr>
<tr>
<td>6. $D/h$</td>
<td>Relative size of plate</td>
</tr>
<tr>
<td>7. $B/h$</td>
<td>Relative scale of channel width</td>
</tr>
<tr>
<td>8. $\Delta$</td>
<td>Relative submerged density</td>
</tr>
</tbody>
</table>
Figure 1. Definition Sketch

Figure 2. Scour Depth versus Reynolds No. (After Shen(1966))

Figure 3. Maximum depth of scour as a function of Froude No.
References


