1.0 INTRODUCTION

In 1987, Nikola Tesla discovered that he could transmit up to 20 MV or more power wirelessly. This was done by sending a signal into upper stratosphere at a frequency of 925 Hz to distances thousands of miles away from the transmitter. Wireless power transfer (WPT) receivers are devices that can wirelessly transmit power to electrical devices. It can be proof by the concept technology that paves the way for charging cell phones, laptops, and many other electronic devices wirelessly. Since then interest in this technology have boomed and it is easy to see why [2].

The Friis transmission equation gives the amount of power an antenna received under ideal conditions from another antenna. So that the Friis equation is important to optimized wireless power transfer. From textbook derivation of this equation, we usually do not understand the physical source of the wavelength-squared term that expresses the effect of diffraction. We quickly learn to use the equation with ease, but without a clear
conceptual understanding of why the equation takes the form that it does.

2.0 CONCEPT OPERATION

The concept of the wireless power device is designed in at least two sections which are the oscillator and antennae coils. These sections are outlined below in Figure 1. The oscillator being used to generate the amount of frequency. An oscillating current is generated at the transceiver circuit and an electromagnetic field is created around the transceiver coil. Due to mutual inductance, a current started to flow in the receiver side. The mutual inductance can be changed when the distance or angle between the coils change. Transmitting signals over long distances requires that the transmission beam have a high degree of directionality and a very large gain requiring a larger size antenna. In the near field there is still the need for directionality and gain, but also the need for the beam not to be affected by outside radiation as it is generally transmitted at lower power levels. For purposes of far field or long distance WPT, wide angle dipole antennas are better. While for near field or short range WPT, helical antennas provide better gain and power transfer.

![Figure 1: Fundamental Interaction](image)

3.0 FRIIS EQUATION

Despite the simplicity of the concept of antenna equivalent area, the dipole derivation found in several popular antenna texts loses some of the important physical meaning in concatenated discussions of infinitesimal dipole radiation resistance, directivity, and the relationship between directivity and effective area. This derivation is reviewed here in a manner reasonably consistent with the notation and terminology in common texts.

By assuming a constant current on a lossless, impedance matched infinitesimal dipole, a radiation integral can be solved to obtain the magnetic vector potential, the curl of which gives the dipole magnetic field. This field in turn produces a dipole electric field through Ampere’s Law. These fields can be used to compute the time-average power density:

\[
\overline{W} = \frac{1}{2} \overline{E} \times \overline{H}^{\ast} = \hat{r} W_r + \hat{\theta} W_{\theta} \quad (1)
\]

where \( \overline{E} \) is the electric field vector, \( \overline{H}^{\ast} \) is the complex conjugate of the magnetic field vector, \( \hat{r} \) is the radially directed unit vector, \( \hat{\theta} \) is the azimuthally directed unit vector, and \( W_r \) and \( W_{\theta} \) are the radial and azimuthal components of complex power density [W/m²]. The radial component of power density is integrated over a closed sphere to obtain the total power flowing in the radial direction, \( P_{\text{rad}} \), which is equated to the power dissipated in the radiation resistance \( R_r \):

\[
P_{\text{rad}} = \eta \frac{\pi}{3} \left| \frac{I_o}{\lambda} \right|^2 = \frac{1}{2} \left| \frac{I_o}{\lambda} \right|^2 R_r \quad (2)
\]

In eq. (1) \( \eta \) is the impedance of free space (\( \approx 120\pi \)), \( I_o \) is the dipole current amplitude, \( I \) is the dipole length, and \( \lambda \) is the wavelength. Eq. (2) is solved for \( R_r \) to obtain:

\[
R_r = \frac{\pi}{3} \left( \frac{I}{\lambda} \right)^2 = 80\pi^3 \left( \frac{I}{\lambda} \right)^2 \quad (3)
\]

the radiation resistance of a lossless, matched infinitesimal dipole (\( I < \lambda \)) in free space.
The effective area of the antenna $A_e$ is defined as the area which multiplied by the incident power density $W_i$ gives the power delivered at the terminals of the antenna $PT$ so that

$$A_e = \frac{P_T}{W_i} = \frac{|I_r| R_T}{W_i}$$

(4)

Assuming the conjugate-matching conditions of maximum power transfer into an antenna with radiation resistance $R_r$ and loss resistance $RL$, the maximum effective area is shown to be

$$A_{em} = \frac{|V_r|^2}{8W_i} \left( \frac{1}{R_r + R_L} \right)$$

(5)

For a lossless antenna $(RL = 0)$ in free space and an incident uniform plane wave with electric field $E$ and power density $W_i = EI/2\eta$, we can use eq. (3) in (5) to write

$$A_{em} = \frac{(EI)^2}{8} \left( \frac{E_i^2}{2\eta} \right) \left( \frac{80\pi^2\lambda^2}{\lambda^2} \right)$$

(6)

This can be algebraically reduced, using $\eta = 120\pi$, to

$$A_{em} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

(7)

The important wavelength-squared dependence of the Friis transmission equation becomes apparent at this point in this derivation, but the physical meaning is often lost for students trying to follow the lengthy process. Having the wavelength squared term come from the radiation resistance equation into the effective area equation does not necessarily provide an intuitive insight into its meaning. At this point we take a short detour to find the directivity of the infinitesimal dipole [3]. The previously derived dipole fields are used, along with knowledge that the product of a power density and range squared gives the radiation intensity $U$ [W/sr], to write

$$U = r^2W_o = \frac{r^2}{2\eta} |E_r|^2 = \eta \left( \frac{1}{2\lambda} \right)^2 \sin^2 \theta$$

(8)

The maximum radiation intensity $U_{\text{max}}$ occurs at $\theta = \pi/2$:

$$U_{\text{max}} = \eta \left( \frac{1}{2\lambda} \right)^2$$

(9)

from which the maximum directivity is found as

$$D_{ao} = \frac{U_{\text{max}}}{U} = 4\pi \frac{P_{\text{rad}}}{P_T} = \frac{4\pi}{\eta} \frac{2}{3\lambda} = \frac{3}{2} = 1.5$$

(10)

The derivation continues with the development of an equation that relates the maximum effective area to the maximum directivity of the infinitesimal dipole. This begins by noting that a directive transmitting antenna radiates a power density at range $R$ equal to the isotropic power density multiplied by the antenna directivity:

$$W_i = W_o D_t = \frac{P_{D_t}}{4\pi R^2}$$

(11)

The received power is therefore

$$P_r = W_i A_r = \frac{P_{D_o} A_r}{4\pi R^2}$$

(12)

where $A_r$ is the effective area of the receiving antenna. If we add subscripts to indicate maximum values of $D$ and $A$, this equation can be rearranged to the form [4].

$$D_{ao} A_{om} = \frac{P_r}{P_i} \left( \frac{4\pi R^2}{D_{ao}} \right)$$

(13)

which by reciprocity can also be written as

$$D_{ao} A_{om} = \frac{P_r}{P_i} \left( \frac{4\pi R^2}{D_{ao}} \right)$$

(14)

By equating (13) and (14), we arrive at

$$\frac{D_{ao}}{A_{om}} = \frac{D_{ao}}{A_{om}}$$

(15)

which says that increasing the directivity of an antenna results in a proportional increase of effective area. An isotropic transmitting antenna would have $D_{ao} = 1$, so its maximum effective area is

$$A_{om} = \frac{A_{em}}{D_{ao}} = \frac{0.119\lambda^2}{1.5} = \frac{\lambda^2}{4\pi}$$

(16)
Therefore, the maximum effective area of an infinitesimal dipole used as a receiving antenna is

\[ A_{\text{em}} = D_{\text{or}} A_{\text{en}} = D_{\text{or}} \left( \frac{\lambda}{4\pi} \right) \]  

(17)

Once the applicability of eq. (17) to all antennas has been accepted, this equation can be used in eq. (12) to write an expression for the power received by a nonisotropic receiving antenna (with multiplicative efficiency terms \( e_r \) and \( e_t \)):

\[ P_r = A W_i = e_r e_t D_t \left( \frac{\lambda^2}{4\pi} \right) \left( \frac{P_t D_t}{4\pi R^2} \right) \]  

(18)

From eq. (18) we can write the Friis transmission equation in terms of a ratio of received to transmitted power, using either antenna directivity or gain \( G \):

\[ \frac{P_r}{P_t} = e_r e_t \left( \frac{\lambda^2}{4\pi R} \right)^2 D_t \left( \frac{\lambda}{4\pi R} \right)^2 G_t G_i \]  

(19)

Thus we arrive at the desired equation, having seen along the way several important antenna terms and relationships, but having also perhaps lost sight of some of the physical significance of the free-space spreading term in parentheses.

Since wavelength and frequency \( f \) are related by the speed of light \( c \), we have the Friis Transmission Formula in terms of frequency [5]. The equation that relates frequency, wavelength and the speed of light

\[ c = f\lambda \]  

(20)

By equating (19) and (20), therefore the Friis Transmission in term of frequency [6]

\[ P_r = \frac{P_t G_t G_i c^2}{(4\pi R f)^2} \]  

(21)

Equation (21) shows that more power is lost at higher frequencies. This is a fundamental result of the Friis Transmission Equation. This means that for antennas with specified gains, the energy transfer will be highest at lower frequencies. The difference between the power received and the power transmitted is known as path loss. Said in a different way, Friis Transmission Equation says that the path loss is higher for higher frequencies.

### 4.0 RESULTS AND DISCUSSION

Based on the data collected from the experiment [1], we obtain the wireless power transmission efficiency by applying the Friis equation. Table 1 and Figure 2 below shows the result. We use to find the constant value based on the experiment and simulation to make the optimization. The constant value is added on small scalar experiment

\[ k = \frac{4\pi R f}{c} \sqrt{\frac{P_t}{P_r G_t G_i}} \]  

(22)

From the calculation, the average constant is \( 6.3746 \times 10^{-11} \)

![Table 1 Varied Frequency Data vs Power Receive Experiment and Simulation](image)

<table>
<thead>
<tr>
<th>( f ) (MHz)</th>
<th>Power Experiment (Watt)</th>
<th>Power Simulation (Watt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.002401</td>
<td>0.106</td>
</tr>
<tr>
<td>0.4</td>
<td>0.038423</td>
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<tr>
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<td>4.905156</td>
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</tr>
<tr>
<td>1.2</td>
<td>5.714286</td>
<td>5.7154</td>
</tr>
<tr>
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</tr>
<tr>
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<tr>
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<td>5.71546</td>
</tr>
<tr>
<td>2.0</td>
<td>5.488</td>
<td>5.59434</td>
</tr>
</tbody>
</table>

From the table data above, the higher frequency will give higher power receive. The optimum frequency for highest power could be transmitted is 1.5MHz.

![Figure 2 Graph of Frequency(MHz) vs Power Receive(Watt)](image)
getting higher as the frequency is higher. The graph also prove that the value obtain from the simulation and the experiment is almost the same.

5.0 CONCLUSION

The Friis transmission equation is a sufficiently simple and practical tool to use. In this work, we can conclude that the experimental and the wireless power transfer algorithm calculation prove that the optimum frequency for receiver power is 1.5MHz. In calculation, constants are treated in several different ways depending on the operation. The reason constant is use in this work is because the data value we obtain is still not same and too big when compare to the experiment value. The constant is help to make the calculation value almost same when compare to the experimental value result.

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References


