PID CONTROLLER TUNING OPTIMIZATION USING
GRADIENT DESCENT TECHNIQUE FOR AN ELECTRO-
HYDRAULIC SERVO SYSTEM

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Graphical abstract

Abstract

The prominent performance of electro-hydraulic servo (EHS) system has received a positive admission in the industrial field. EHS system is well known to be disclosed to the parameter variations, disturbances and uncertainties which are affects by the changes in the operating conditions such as friction, internal and external leakage. The complexity and nonlinear characteristic of the EHS system leads to a great challenge in controller development and system modelling. The performance of the utilized controller can be improved in order to achieve its best capability. In this paper, the basic knowledge in optimization of the proportional-integral-derivative (PID) controller through Gradient Descent (GD) method was discussed. The PID parameters obtained through Ziegler-Nichols (ZN) tuning method has been optimized using the GD method via MATLAB/Simulink software. The findings illustrate significant improvement in the positioning tracking performance by applying the developed optimization technique. Therefore, the issues that were degraded the EHS system performance have been reduced.

Keywords: Electro-hydraulic servo system; Gradient Descent optimization technique; PID controller; position tracking

Abstrak

Lonjakan prestasi yang dihasilkan oleh sistem servos elektro-hidraulik (EHS) telah mendapat sambutan positif di dalam bidang industri. Sistem EHS seperti yang telah diketahui telah terdedah dengan perubahan parameter, gangguan dan ketidaktentuan yang terdapat di dalam sistem tersebut yang disebabkan oleh perubahan semasa operasi seperti geseran, kebocoran dalam mahupun luaran. Kerumitan dan ciri tak linear sistem EHS telah membawa cabaran yang sukar dalam pembangunan pengawal dan pemodelan terhadap sistem tersebut. Prestasi untuk pengawal yang digunakan boleh ditingkatkan bagi mencapai kemampuan yang terbaik. Di dalam kertas ini, pengetahuan asas dalam pengoptimuman pengawal berkadar-kamiran-terbitan dengan menggunakan kaedah keturunan kecerunan (GD) telah dibincangkan. Parameter pengawal PID yang telah diperoleh melalui kaedah Ziegler-Nichols (ZN) telah dioptimumkan dengan menggunakan kaedah GD melalui persian MATLAB/Simulink. Penemuan menunjukkan peningkatan prestasi yang ketara untuk prestasi penjejakan kedudukan dengan menggunakan kaedah pengoptimuman yang telah dibangunkan. Oleh itu, isu-isu yang boleh membawa kesan kepada penurunan prestasi sistem EHS dapat dikurangkan.

Kata kunci: Sistem servos elektro-hidraulik; teknik pengoptimuman keturunan kecerunan; pengawal PID; penjejakan kedudukan

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1.0 INTRODUCTION

Due to the ability to create a large force, torques, high energy density, hydraulic system becomes widely used in design industrial or mobile machines for the past decades [1]. The power sources such as electric motor or engine drives provides hydraulic pump dynamics that deliver fluid under pressure. The fluid delivered through pressure is used to create the necessary movements in linear, rotary or reciprocating [2]. The movement is the desired end function such as, lifting, pressing, clamping, and orienting.

Electro-hydraulic servo (EHS) system has appealed a great attention as an actuator in heavy engineering applications due to its capability in provides high forces with compact design [3]. By utilizing the important properties of EHS system, many heavy-duty works had accomplished. The advanced design of EHS system with the versatility of electronic and hydraulic components offers a massive enhancement for those applications performance [4].

However, the EHS system has known to be bothered by a nonlinearities problem that affected by internal and external disturbances or uncertainties that degrades the performances of EHS system [5]. These problems simultaneously influence the robustness and position tracking accuracy [6]. The roots of these nonlinearities problems are an internal frictions as well as internal leakage in the hydraulic actuation system.

Many studies related to the electro-hydraulic servo (EHS) system problems have been conducted to figure out right direction to surmount these problems. One of the ways is by optimizes the system controller performance. As the optimization technique becomes popular nowadays, it can be utilized to optimize various types of controller such as proportional-integral-derivative (PID) controller that employed in this paper [7].

In order to optimize the PID controller, the PID parameter was first obtained by using Ziegler-Nichols (ZN) tuning methods. Ziegler and Nichols published a paper that suggested a rule for tuning PID controller through the experimental step response or by adjusting the value of that result in marginal stability [8]. After the obtained parameter inserted into the controller, the improvement can be achieved by utilizing optimization technique to the controller.

Optimization is described as the cognitive operation of researching for the solution that is more useful among the solutions. This condition implies that an outcome of using optimization technique to the problem or design must yield numbers that will define our solution [9]. Gradient descent (GD) is one of the methods in optimization that is a first-order optimization algorithm. The local minimum of a function can be found by using GD, which by taking the steps that are proportional to the negative of the gradient or approximate to the negative gradient of the function at the current point. If taking a step proportional to positive of the gradient, the solution will be approached to local maximum of that function. The associated procedure is then known as gradient ascent [10]. Many excellent optimization methods had proposed in different systems, such as particle swarm optimization (PSO), and clonal selection algorithm method. These optimization methods are typically a complex algorithm with a long execution time. By relevant to the real-time characteristic, these methods are too complicated and time-consuming that cannot meet the demands [11]. Therefore, in this paper, GD method is proposed to correct the errors.

In this paper, the performance of position tracking control for EHS system will be investigated using a PID controller with optimization technique. The servo valve and hydraulic actuator integrating with nonlinear dynamics model are derived. Subsequently, the performance of position tracking controller is compared with the optimized controller performance in order to demonstrate the significant enhancement of the controller through the proposed technique.

2.0 MODELLING EHS SYSTEM

Dynamics equation of an EHS system consists of hydraulic cylinder connected to the servo valve through the pipeline as shown in Figure 1. The displacement of cylinder actuator will be generated by utilized the servo valve that will regulated the oil flow from cylinder chamber to the hydraulic cylinder. The spring and damper that is attached to the mass will generate the counter force against cylinder actuator [12].

By producing mechanical motion of the spool valve, the electrical current is supplied to the coil that connected to the servo valve. The torque motor that received the power source will drive the servo spool valve to the desired position. The torque motor electrical signal is given as in equation (1), [13].

$$\frac{dl}{dt} = L_c i_c + R_c i_c$$  \hspace{1cm} (1)

Where $R_c$ and $L_c$ are the coil resistance and inductance respectively.

The dynamics of the servo valve are represents by a second order differential equation that related to electric current drive from the torque motor as expressed in (2).
\[ \frac{d^2 x_v}{dt^2} + 2 \xi \omega_n \frac{dx_v}{dt} + \omega_n^2 x_v = \dot{x}_w \] (2)

where \( \xi \) is the damping ratio, while \( \omega_n \) is the natural frequency of servo valve.

In servo valve mechanical design, the spool valve is unexposed from flow leakages and dead-zone problems for each port. Critical centred on the spool valve is considered. The flow \( Q \) in each of the chambers controlled by servo valve can be modelled from the orifice equations relates the spool valve displacement \( x_v \) and pressure difference \( P_v \). The ideal orifice equation is written in (3).

\[ Q = k x_v \sqrt{\Delta P_v} \] (3)

The flow relations that neglect the internal leakages in servo valve for each chamber are given in (4) and (5).

\[ Q_1 = \begin{cases} K_1 x_v \sqrt{P_1 - P_s} & ; x_v \geq 0, \\ K_1 x_v \sqrt{P_s - P_1} & ; x_v < 0, \end{cases} \] (4)

\[ Q_2 = \begin{cases} -K_2 x_v \sqrt{P_2 - P_s} & ; x_v \geq 0, \\ -K_2 x_v \sqrt{P_s - P_2} & ; x_v < 0, \end{cases} \] (5)

The hydraulic actuator volumes for each chamber are modelled in (6) and (7).

\[ V_1 = V_{\text{ine}} + A_p (x_1 + x_p) \] (6)

\[ V_2 = V_{\text{ine}} + A_p (x_1 - x_p) \] (7)

where \( V_{\text{ine}} \) is the volume between hydraulic cylinder and pipeline.

Pressure for each chamber can be obtained by defines the relation between bulk modulus, volume, and flow rate as expressed in (8) and (9).

\[ P_1 = \frac{\beta}{V_{\text{ine}} + A_p (x_1 + x_p)} \left[ (Q_1 - q_{12} - q_1 - \frac{dV_1}{dt}) \right] \] (8)

\[ P_2 = \frac{\beta}{V_{\text{ine}} + A_p (x_1 - x_p)} \left[ (\frac{dV_2}{dt} - Q_2 - q_{21} - q_2) \right] \] (9)

Through the overall dynamics equation of moving mass, damper, and spring, the total force produced from hydraulic actuator can be obtained in (10).

\[ F_p = A_p (P_1 - P_2) = M_p \frac{d^2 x_p}{dt^2} + B_s \frac{dx_p}{dt} + K_s x_p + F_f \] (10)

In a simulation study, the parameters used in a nonlinear model of EHS system have been tabulated in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_c )</td>
<td>Servo-valve coil resistance</td>
<td>100 ( \Omega )</td>
</tr>
<tr>
<td>( L_c )</td>
<td>Servo-valve coil inductance</td>
<td>0.59 H</td>
</tr>
<tr>
<td>( I_{\text{sat}} )</td>
<td>Torque motor saturation current</td>
<td>0.02 A</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Servo-valve damping ratio</td>
<td>0.48</td>
</tr>
<tr>
<td>( \omega_n )</td>
<td>Servo-valve natural frequency</td>
<td>543 rad/s</td>
</tr>
<tr>
<td>( K )</td>
<td>Servo-valve gain</td>
<td>2.38x10^5 ( \text{m} \text{s}^{-2} \text{kg}^{-1/2} )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Hydraulic fluid bulk modulus</td>
<td>1.4x10^5 ( \text{N} \text{m}^{-2} )</td>
</tr>
<tr>
<td>( P_s )</td>
<td>Pump pressure</td>
<td>2.1x10^7 Pa</td>
</tr>
<tr>
<td>( P_r )</td>
<td>Return pressure</td>
<td>0 Pa</td>
</tr>
<tr>
<td>( K_s )</td>
<td>Spring stiffness</td>
<td>10 Nm</td>
</tr>
<tr>
<td>( X_s )</td>
<td>Total actuator displacement</td>
<td>0.1 m</td>
</tr>
<tr>
<td>( A_p )</td>
<td>Piston area</td>
<td>645x10^{-6} m^2</td>
</tr>
<tr>
<td>( M_p )</td>
<td>Total mass</td>
<td>9 kg</td>
</tr>
<tr>
<td>( B_s )</td>
<td>Damping coefficient</td>
<td>2000 Ns/m</td>
</tr>
</tbody>
</table>

3.0 METHODOLOGY

3.1 Ziegler-Nichols (ZN) PID Tuning Method

Simulation works will be conducted by applying these parameters as tabulated in Table 1 into the mathematical equation of the EHS system. The Simulink block diagram of an EHS system without concerning the internal leakage and external leakage of the system as describe from the equation (1)-(10) is shown in the Figure 2.

PID controller is then connected to the models to control the displacement of the cylinder actuator according to the desired reference input. The parameter values for \( K_p, K_i, \) and \( K_d \) was first obtained through the ZN tuning method [14].
The PID controller commonly enveloped with three important controller terms provided treatment of transient as well as steady state responses. The transfer function of the PID controller is usually expressed as:

\[ C(s) = K_p + \frac{K_i}{s} + K_d s = K_p (1 + \frac{1}{\tau_i s} + T_d s) \]  \hspace{1cm} (11)

where \( K_p \) is the proportional gain, \( K_i \) is the integral gain and \( K_d \) is the derivative gain. The proportional terms provide rapid control to the transient response proportional to the possible steady state error signal while the integral terms reduce or eliminate steady state errors through low frequency compensation. The derivative terms improving transient response through high frequency compensation.

By considering the ZN tuning method procedure, \( K_i \) and \( K_d \) is first reduce to 0. The value of \( K_p \) is then increased to the ultimate gain value \( K_u \) where sustained oscillations is achieved as depicted in Figure 3. Corresponding sustained oscillations period \( T_u \) can be obtained by measuring the period within full wave cycle [16].

The controller gains are then obtained by applying the ultimate gains value and corresponding sustained oscillations period to the formula as tabulated in Table 2.

### Table 2 PID controller formula [14]

<table>
<thead>
<tr>
<th>PID Type</th>
<th>( K_p )</th>
<th>( T_i )</th>
<th>( T_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.5 ( K_u )</td>
<td>Inf</td>
<td>0</td>
</tr>
<tr>
<td>PI</td>
<td>0.45 ( K_u )</td>
<td>( T_i/1.2 )</td>
<td>0</td>
</tr>
<tr>
<td>PID</td>
<td>0.6 ( K_u )</td>
<td>( T_i/2 )</td>
<td>( T_i/8 )</td>
</tr>
</tbody>
</table>

3.2 Gradient Descent Optimization

The methods of steepest descent, GD is an algorithm applied to obtain a minimum point for the particular function. Conversely, gradient ascent is an algorithm that used to find a maximum point nearer to the current result. In any starting point of the function, GD algorithm shifting the solution to the negative direction of the gradient to reaching minimum point as indicated in Figure 4. The function value will be diminished at the fastest rate when running along the slope direction. The negative sign of the gradient vector indicates the direction of the steepest descent. Hence, the GD method is expected to arrive at the minimum point faster than other non-gradient based optimization methods [17]. The iteration process had repeated according to the equation (12):

\[ X_{i+1} = X_i - \lambda_i \nabla f(X_i) = X_i - \lambda_i g(X_i) \]  \hspace{1cm} (12)

at which the \( \lambda_i > 0 \) satisfies:

\[ f(X_i - \lambda_i g(X_i)) = \min_{\lambda_i > 0} f(X_i - \lambda_i g(X_i)) \]  \hspace{1cm} (13)

where \( \lambda \) denotes the step size, and gradient operator \( \nabla \) of the function \( f(X) \). While \( g(X_i) \) is the gradient at the current point.

By moving to the point where function \( f \) taking on a minimum value, the directional derivative is given by:

\[ \frac{d}{d \lambda_i} f(X_{i+1}) = \nabla f(X_{i+1})^T \cdot \frac{d}{d \lambda_i} X_{i+1} = -\nabla f(X_{i+1})^T g(X_i) \]  \hspace{1cm} (14)

Let the equation (14) equal to zero, and applying the step size \( \lambda \) to the function \( \nabla f(X_{i+1}) \) and \( g(X_i) \), the directional derivative of the function will be obtained in orthogonal form or in the zigzag pattern as illustrated in Figure 4.

The \( \lambda > 0 \) is a minor value that leads a small step to the function. An appropriate value for the \( \lambda \) is very
significant, smaller value could increase convergence time and higher value may lead to diverging. The appropriate value of \( \lambda \) yield to stable condition as:

\[
f(X_{i+1}) \leq f(X_i)
\]

(15)

Figure 4 The direction to reaching local minimum

Alternatively, algorithm can be started by chosen a fix value for step size \( \lambda \). The step size \( \lambda \) will need to be regulated if necessary to make sure the function decreases at each iteration [18].

The proposed controller parameter will be transformed until the stopping criteria met. These criteria included the variation of the parameters, the variation of the gradient, the function reached lower bound, the Euclidean norm of the gradient and the fixed maximum for the number of iterations [19]. The best parameters hence achieved when all the criteria of GD is fulfilled.

4.0 RESULTS AND DISCUSSION

ZN tuning method had produced the significant PID parameter value that could later be improve in order to obtain the more better performance PID controller parameter. Through the ZN tuning procedure, the parameter that fed into the EHS system with a step input reference signal yield the output waveform as depicted in Figure 5 (thin line). Obviously, the signal has overshoot before reaching steady state condition. This condition could be very harmful if applied to the real applications. Hence, the controller with no overshoot or a minimum overshoot with fast settling time is a target for every type of controller. By utilize optimization technique based GD method to the controller, the overshoot of the PID controller have been eliminated (thick line). Figure 5 depicts the waveform before and after the optimization process.

Two iterations have been took to complete the optimization process with the detail as shows in Table 3 when the optimization solver (Gradient Descent, fmincon) had launched a result that met the design requirements within the parameter bounds and tolerances. In each of optimization iteration, the algorithm executes the simulation while an optimization solver (Gradient Descent, fmincon) modified the design variables to reduce the distance between the line segment design requirement and the simulated response. The iteration is the number for optimization solver attempt to evaluating the objective function and constraint. The F-count is a header in the iterative display for many solvers. All the attempt steps increase the F-count by one at the nearby point, regarding to the algorithm (12) to (15) indicated in section 3.2. Then, the Check Step Response Characteristic indicate the result according to the constraint of piecewise linear bounds illustrated in (13).

Table 3 The detail of an Optimization Process

<table>
<thead>
<tr>
<th>Iteration</th>
<th>F-count</th>
<th>Check Step Response Characteristic</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>0.19</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>-0.02</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>-0.02</td>
</tr>
</tbody>
</table>
The comparison results for the PID controller and optimized PID controller for position tracking performance is shown in Figure 6 and 7. In both figures, step input and sinusoidal input that fed to the system respectively. The simulation result illustrates better performance through optimized parameter that fed into the PID controller. Figure 6 depicts the optimized controller produced a much stable response and eliminated the overshoot effect of the response. The blue line denotes the step input reference signal, the red line represent the output signal through the ZN tuning parameter and the light blue line indicates the parameter that is optimized based GD method.

The line with same colour has been applied to the sinusoidal input reference signal as depicted in Figure 7. As shown in the zoomed in figure, GD output had provides much closer result to the reference signal compared to the ZN output. To provide a much clear view to the gap in between ZN and GD output, Table 4 provided the Root Mean Square Error (RMSE) analysis of ZN and GD to the reference signal. The result indicated the significant changes which are 26.67% as compared with ZN tuning method.

![Figure 6: The comparison result for step input](image1)

![Figure 7: The comparison result for sinusoidal input](image2)

### Table 4  RMSE analysis

<table>
<thead>
<tr>
<th>Method</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZN</td>
<td>0.0015</td>
</tr>
<tr>
<td>GD</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

The improvement is vital in order to enhance the accurateness and consistency of the system controller. The detail of these two tuning methods evaluated in step and sinusoidal input are tabulated in Table 5.

### Table 5  Parameters value for PID controller and Optimized PID Controller

<table>
<thead>
<tr>
<th>PID</th>
<th>Parameters Value</th>
<th>Gradient Descent Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ziegler-Nichols</td>
<td>Step Input</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sine input</td>
</tr>
<tr>
<td>$K_p$</td>
<td>1020</td>
<td>289.6602</td>
</tr>
<tr>
<td>$K_i$</td>
<td>0.0150</td>
<td>1487.6219</td>
</tr>
<tr>
<td>$K_d$</td>
<td>0.0038</td>
<td>2.4999</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0037</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0003</td>
</tr>
</tbody>
</table>

### 5.0 CONCLUSION

In this paper, mathematical modelling of EHS system has been derived and implemented in the simulation study. The performance of the PID controller has been evaluated by consider the GD technique that has been applied to the controller. The numerical simulation shows that an optimization technique provides significant improvement to the controller, and produced much precise position trajectory tracking.

**Acknowledgement**

The authors would like to thank to the University Teknikal Malaysia Melaka (UTeM) and Ministry of Education (MOE) for their support. The research was...
References


