INVESTIGATION ON OPTICAL FIBRE CONFIGURATIONS FOR PROCESS TOMOGRAPHY

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Abstract. This paper describes an investigation on the optimum optical fibre sensing configurations to be utilised for a tomography process system. The response of the sensors was based on two models: (1) optical attenuation model due to material with differing optical densities, and (2) optical path length model which is used for opaque solids in air or liquid. The circular flow pipe is mapped onto an eight by eight rectangular array of pixels. An investigation is carried out on four flow models: a single pixel flow, two pixels flow, half flow and full flow models. Various arrangements of the fibre sensors are considered for different projections. The forward problem is solved for all combinations of flow model and sensor arrangement. Linear back projection (LBP) algorithm is used to reconstruct the image. A modified reconstruction algorithm combining LBP and a priori knowledge is outlined. Numerical tests, based on the reconstruction, are used to identify the best system to be implemented in hardware.

Keywords: Optical Fibres, Process, Tomography, Modelling, Sensor

1.0 INTRODUCTION

Analysing the internal characteristics of a process plant via direct means is important in order to improve the design and operation of production and control equipment. The measuring instruments must employ robust, non-intrusive sensors in such appli-
cations which can be employed in aggressive and fast moving fluids and multi-phase mixtures.

Until now, there exists various types of flow meters utilising a range of different techniques. The accuracy and functionality of these meters depend on the flow profile. To enable these techniques to work, prior assumptions of the flow distribution have to be made as the instruments themselves are not capable of providing local information on the flow. Tomography provide a means of viewing the internal characteristics of a flow region from which local information such as volume fraction and velocity can be extracted. It provides more accurate means of computing flow rate and eliminate the uncertainty in measurement due to the inability to predict flow profile related to the existing flow measurement methods [1].

This paper presents an investigation on the use of tomographic measurement for on-line monitoring of particles and droplets having low concentration being conveyed by a fluid. A typical example is the measurement of crude oil being discharged by tankers flushing their oil storage tanks. This project aims to combine the sensitivity of optical sensors with the area monitoring potential of process tomography.

2.0 MATHEMATICAL MODELLING

In order to predict the spatial and temporal behaviour of a process, modelling is carried out and it becomes more significant as the inherent complexity of a process increases [2]. Three important stages are involved in modelling the sensor configurations:

(1) Identify the mathematical model of the sensor, and determine the associated equations and boundary conditions.

(2) Establish the geometric model of the sensor taking into account of the significant aspects and special features of the problem domain so as to minimize the amount of data.

(3) Select an efficient numerical method in order to realize a computer solution of the problem.

The forward problem for the individual sensors is modelled, used to solve the inverse problem and derive the linear back projection and filtered back projection algorithms [3]. Modelling is carried out based on two parameters affecting the measured output of the sensor:

(1) Path length of the sensing beam within the pipeline projections [4].

(2) Optical attenuation due to changes in optical density within the pipeline.

In previous projects, Ruzairi [5] concentrated on the path length method whereas Ramli [6] focused effort on the optical attenuation method. This project investigates both methods, which enables a comparison between their performance to be made.
2.1 Projection Geometry

Various arrangements of the fibre sensors are considered for different types of projections. These include:

(a) two orthogonal projections consisting of several parallel views (Figure 1).
(b) two rectilinear projections consisting of several parallel views inclined at 45° to one another (Figure 2).
(c) a combination of two orthogonal and two rectilinear projections (Figure 3).
(d) three fan-beam projections (Figure 4).
(e) four fan-beam projections (Figure 5).

Figure 1  Orthogonal projections

Figure 2  Rectilinear projections

Figure 3  Combination of two orthogonal and two rectilinear projections

Figure 4  Three fan-beam projections
In Figures 1 to 5, $s_i$ stands for sensor $i$ and $p_k$ stands for projection $k$. For example; $s_{21}, p_3$ corresponds to sensor 21 which is located in the third projection.

For the two orthogonal projections, each projection provides eight light beams (views) which are parallel and equally spaced to each other. The light emitters and detectors are arranged in a one-to-one basis i.e. each emitter has a corresponding detector. Similarly in the rectilinear projections, the emitters and detectors are also positioned on a one-to-one basis. In the case of the two rectilinear projections, the projections are inclined at $45^\circ$ to the horizontal whereas in the case of the combination of one orthogonal and two rectilinear projections several parallel views are inclined at $120^\circ$ to one another.

In Figures 4 and 5, a series of angular projections of the light source and detectors are used to interrogate the measurement section; these are termed fan beam projections. In the case of three fan-beam projections, three light sources are used. Each light source will supply twelve light beams which are spaced at $10^\circ$ intervals. This results in the cross-section of the pipe being interrogated by a total of thirty six light beams as shown in Figure 4. In the case of four fan-beam projections, four light sources are utilised and the projections are placed $90^\circ$ apart as shown in Figure 5 resulting in a total of 48 light beams.

### 3.0 OPTICAL PATH LENGTH MODEL

The optical path length model is based on the length of the optical sensing beam within the conveyor in which the greater the active length, the greater the probability of a particle intercepting the beam [5]. The voltage of each individual path length sensor increases with increased particle flow rate or in other words the more particles that intersect a light beam the greater the voltage. It is assumed that the relationship
between the number of particles passing through a beam and the corresponding sensor output voltage is linear.

### 3.1 Forward Problem for Path Length Method

The theoretical output of each sensor under no-flow and flow conditions when the sensing area is considered to be two dimensional is provided by the forward problem. By solving the forward problem a series of sensitivity matrices is generated. Each matrix is associated with a specific sensor and relates to the sensor output under flow conditions.

To facilitate the solution of the forward problem, several simplifications and assumptions are made. It is assumed that light beams travel in straight lines and that for each light beam, the distance between emitter and detector is 100 mm and the beam width is 1 mm. This means that the model neglects beam spread. It is also assumed that the beam in each pixel has a rectangular shape which simplifies the calculation of the area of the light beam in each pixel. Each pixel is designated as \( P_{ij} \), where \( i \) is the row number and \( j \) is the column number. The dimension of each pixel is 10 mm \( \times \) 10 mm.

Computation of the area of each pixel enclosed by the circle (flow pipe) is carried out. Then the area within a specific light beam in each pixel is obtained. Pixels outside the flow pipe (represented by a circle) and pixels in which the specified light beam does not pass are assumed to contain air. This \( a \ priori \) knowledge enabled all such pixels to be assigned zero sensitivity values [5]. The sensitivity matrix for each light beam (or for each sensor) is formed by calculating the ratio of the area of the light beam in each pixel to the area of the corresponding pixel. Each pixel is evaluated separately and the contribution from each pixel forms the sensitivity map [5]. Each sensor is considered separately.

### 3.2 Inverse Problem for the Path Length Model

The distribution of material within the pipe which provides the measured sensor outputs is estimated using the inverse problem. As the number of projections (i.e. 36 for the three projection system) is limited and the number of pixels inside the pipe (i.e. 60) is larger than the number of projections, an analytic solution is not possible and as such linear back projection (LBP) is used to solve the inverse problem.

#### 3.2.1 Linear Back Projection

In this algorithm, the concentration profile is obtained by combining the voltage reading from each sensor (measured data) with its computed sensitivity map. By multiplying each sensitivity map matrix with its corresponding sensor reading, a tomographic image can be reconstructed. This results in \( n \times 8 \times 8 \) matrices, where \( n \) is the number of projections. Mathematically, this process can be expressed as
\[ V_{ij} = \sum_{n=1}^{n=36} V_{jn} s_{jn} \]  

(1)

where

\[ V_{ij} = \text{voltage distribution in } 8 \times 8 \text{ matrix} \]
\[ V_{jn} = \text{voltage for } n\text{th sensor} \]
\[ s_{jn} = \text{sensitivity map for } n\text{th sensor in the form of an } 8 \times 8 \text{ matrix}. \]

The voltage distributions, \( V_{ij} \) should be converted to concentration values. However, this has not been done for this paper as it only requires rescaling. The voltage distributions are available as matrices (useful for quantitative information) and pictorially. Selected examples are shown in Figures 6 – 10.

These results are discussed in section 5.

**Figure 6**  LBP for two orthogonal projections: half flow model

**Figure 7**  LBP for a combination of two orthogonal and two rectilinear projections: half flow model

**Figure 8**  LBP for three fan-beam projections: half flow model

**Figure 9**  LBP for two rectilinear projections: half flow model
In the optical attenuation model the output value of the sensor is a function of the medium through which the light traverses coming from the emitter. Scattering and beam diversion are neglected in this model. The medium within the process vessel absorbed the light beam as expressed by equation (2)

\[ V_m = V_{in} \exp \left( -\alpha_a x l_a + l_o (\alpha_a - \alpha_o) \right) \]  

where:

- \( V_m \) = voltage of the receiving sensor
- \( V_{in} \) = voltage of the receiver when there is no beam attenuation
- \( \alpha_a \) = absorption coefficient of air
- \( \alpha_o \) = absorption coefficient of object
- \( l_a \) = path length of air
- \( l_o \) = path length of object

Based on the previous research carried out by Ramli [6] and Hartley et al. [7], the attenuation coefficient of air, \( \alpha_a = 0.0142 \text{ mm}^{-1} \) and the attenuation coefficient of the object, \( \alpha_o = 0.05 \text{ mm}^{-1} \). The output voltage for each sensor is calculated for each pixel in turn. This process is repeated until all pixels within the pipe have been considered. This process is also repeated for all the sensors resulting in \( n \times 8 \times 8 \) matrices where \( n \) is the number of views. The linear back projection calculations for the optical attenuation model are similar to the path length model (section 3). Typical results are shown in Figures 11 to 13.

Results for the optical attenuation model are discussed in section 5.
5.0 DISCUSSION OF RESULTS OBTAINED WITH LBP AND FLBP

The complete set of results for the path length modelling for the half flow model are shown in Table 1.

By adding all the voltages in the voltage concentration matrix and dividing the sum by the ideal voltage the error for each reconstructed model was computed. Similar values were obtained for the attenuation model. It is expected that by increasing the number of projections more accurate estimates of the models will be obtained. Nevertheless this do not occur. Some of the large errors obtained using the back projection algorithm are due to the significant smearing effect in the image due to this type of reconstruction. However, another type of error is introduced if all the sensors do not provide the same weighting in the calculations.

This was very noticeable with the fan beam systems, where pixels close to the light sources are more heavily weighted than those further away. This suggests that orthogonal and rectilinear projections should be used. The errors are still unacceptably high. This lead to the hybrid reconstruction algorithm.
Optical sensors are categorised as hard field sensors and so the material in the flow is assumed only to vary the intensity of the received signal. This enables a priori knowledge from the optical sensors to be used in the reconstruction. For an optical sensor when no objects block the path from transmitter to receiver the sensor will produce a zero output value, neglecting the effect of noise inherent in the system.

This is taken into account in the development of a hybrid reconstruction algorithm which incorporates both a priori knowledge and LBP in order to improve the accuracy of the image reconstruction. This algorithm is being developed using the C programming language.

The algorithm is designed for a two, three or four projection systems based on orthogonal and rectilinear projections. The algorithm assumes binary values from the sensors, either zero for no material or one for the presence of material. All pixels associated with sensors indicating zero are set and held at zero for the rest of the calculation. Briefly, the steps involved in the algorithm are:-

1. Generate the usual sensitivity maps for the horizontal, vertical and diagonal sensors.
2. Initialise all sensors to zero.

### Table 1: Estimated reconstruction errors: path length model

<table>
<thead>
<tr>
<th>Projections</th>
<th>Algorithm</th>
<th>No. of Sensors</th>
<th>Error % (no threshold)</th>
<th>Error % Th &gt; 0.5 × peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Orthogonal</td>
<td>LBP</td>
<td>16</td>
<td>3.5</td>
<td>-25.3</td>
</tr>
<tr>
<td>2 Orthogonal</td>
<td>Hybrid</td>
<td>16</td>
<td>-22.4</td>
<td>-25.3</td>
</tr>
<tr>
<td>2 Rectilinear</td>
<td>LBP</td>
<td>22</td>
<td>7.4</td>
<td>-22.3</td>
</tr>
<tr>
<td>2 Rectilinear/2 Rectilinear</td>
<td>Hybrid</td>
<td>22</td>
<td>-31.1</td>
<td>-37.1</td>
</tr>
<tr>
<td>2 Orthogonal/2 Rectilinear</td>
<td>LBP</td>
<td>38</td>
<td>17.5</td>
<td>-21.8</td>
</tr>
<tr>
<td>2 Orthogonal/1 Orthogonal</td>
<td>Hybrid</td>
<td>38</td>
<td>-19.6</td>
<td>-25.3</td>
</tr>
<tr>
<td>2 Rectilinear/1 Orthogonal</td>
<td>LBP</td>
<td>24</td>
<td>26.2</td>
<td>11.8</td>
</tr>
<tr>
<td>2 Rectilinear</td>
<td>Hybrid</td>
<td>24</td>
<td>-22.2</td>
<td>-25.1</td>
</tr>
<tr>
<td>3 fan-beam</td>
<td>LBP</td>
<td>36</td>
<td>23.1</td>
<td>-60.9</td>
</tr>
<tr>
<td>4 fan-beam</td>
<td>LBP</td>
<td>48</td>
<td>63.0</td>
<td>-81.0</td>
</tr>
</tbody>
</table>

### 6.0 HYBRID RECONSTRUCTION ALGORITHM

Optical sensors are categorised as hard field sensors and so the material in the flow is assumed only to vary the intensity of the received signal. This enables a priori knowledge from the optical sensors to be used in the reconstruction. For an optical sensor when no objects block the path from transmitter to receiver the sensor will produce a zero output value, neglecting the effect of noise inherent in the system.

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1. Generate the usual sensitivity maps for the horizontal, vertical and diagonal sensors.
2. Initialise all sensors to zero.
(3) Read in each sensor value.
(4) If the sensor reading = 0, then any pixels traversed by that sensor’s beam are set to zero and omitted from further calculations.
(5) Execute Linear Back Projection.

This simplified algorithm resulted in the significant improvement in the measured errors. The flow chart representing the steps involved in the algorithm is shown in Figure 14.

![Flow chart for the hybrid reconstruction algorithm](image)

This algorithm results in significant improvements in the measured errors. For two rectilinear projections the single pixel, half and full flow models are perfectly recovered i.e. accuracy is 100%. However, with only two projections the two pixels model results in aliasing and four pixel images are produced (Figure 15). With the combination of two orthogonal and two rectilinear projections system all flow all flow models are fully reproduced (Figure 16).

7.0 CONCLUSIONS
Investigation on the optimum optical sensing configuration has been carried out using four flow models. The hybrid reconstruction algorithm is only suitable for hard field sensors. It is unlikely to improve image reconstruction for flow regimes with a void in the pipe centre, such as annular flow. However, optical sensors are intended for use where the conveyed component ratio is less than 10% vol./vol. In this type of convey-
ing, the material being monitored is well dispersed and The design study presented in this paper enabled an optical tomography system to be built practically with four projections; two orthogonal and two rectilinear. Such a system enabled the smearing effect to be removed effectively. The conditioned sensor outputs will be sampled as digital integers so that the calculations are simplified and high speed is more easily obtained. The data will be collected and processed on line using a modification of the hybrid reconstruction algorithm out-lined in the paper.

REFERENCES