UNSTEADY MHD FLOW OF SOME NANOFLOIDS PAST AN ACCELERATED VERTICAL PLATE EMBEDDED IN A POROUS MEDIUM

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Abstract

The present paper deals with the unsteady magnetohydrodynamics (MHD) flow and heat transfer of some nanofluids past an accelerating infinite vertical plate in a porous medium. Water as conventional base fluid containing three different types of nanoparticles such as copper (Cu), aluminum oxide (Al\textsubscript{2}O\textsubscript{3}) and titanium oxide (TiO\textsubscript{2}) are considered. By using suitable transformations, the governing partial differential equations corresponding to the momentum and energy are converted into linear ordinary differential equations. Exact solutions of these equations are obtained with the Laplace Transform method. The influence of pertinent parameters on the fluid motion is graphically underlined. It is found that the temperature of Cu-water is higher than those of Al\textsubscript{2}O\textsubscript{3}-water and TiO\textsubscript{2}-water nanofluids.

Keywords: MHD flow, nanofluid, accelerating plate, porous medium

Abstrak

Karya ini berkaitan dengan aliran magnetohidrodinamik (MHD) tak mantap dan pemindahan haba bagi sesetengah bendalir nano melepasi plat menegak tak terhingga memecut tak terhingga dalam medium berliang. Air sebagai bendalir lazim asas mengandungi tiga jenis nano zarah berbeza iaitu kuprum (Cu), aluminium oksida (Al\textsubscript{2}O\textsubscript{3}) dan titanium oksida (TiO\textsubscript{2}) dipertimbangkan. Dengan menggunakan penjelmaan yang sesuai, persamaan menakluk pembezaan separa berkaitan dengan momentum dan tenaga di gubah kepada persamaan pembezaan linear biasa. Penyelesaian tepat bagi persamaan ini dipertengah dengan kaedah penjelmaan Laplace. Pengaruh bagi parameter berkaitan terhadap pergerakan bendalir digariskan secara grafik. Didapati bahawa suhu air-Cu lebih tinggi daripada bendalir nano bagi air-Al\textsubscript{2}O\textsubscript{3} dan air-TiO\textsubscript{2}.

Kata kunci: Aliran MHD, bendalir nano, plat memecut, medium berliang

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1.0 INTRODUCTION

The heat transfer fluids such as water, engine oil and ethylene glycol have limited heat transfer capabilities due to their low thermal conductivity. Different ways have been used to increase the convective heat transfer performance of the these fluids such as changing flow geometry, boundary conditions, or by increasing thermal conductivity. It is also true that metals have higher thermal conductivities than fluids. Thermal conductivity can be increased by the adding metals to the base fluids. The resultant fluids are termed as nanofluids. This classical idea was first introduced by Choi [1]. Nanofluids are solid-liquid...
composite materials consisting of solid nanoparticles (or nanofibers with sizes typically of 1–100 nm) suspended in liquid. Actually nanofluids are the homogenous mixture of base fluid and nanoparticles. There are number of common base fluids including water, organic liquids (e.g. ethylene, tri-ethylene-glycols, refrigerants), oil and lubricants, bio-fluids, polymeric solution and other common liquids. After the first work of Chai [1], many other researchers have made their useful investigations that involve the nanoparticles. Buongiorno [2] established the conservation equations of nanofluids based on thermophoresis and Brownian diffusion factors. Kuznetsov and Nield [3] extended the classical model of Cheng and Minkowycz [4] by incorporating the effects of Brownian motion and thermophoresis. After the success of these two models, many other researchers have used these models in their own convective heat transfer problems [5-10].

The study of magnetohydrodynamic (MHD) flow is very important because the influence of a magnetic field on the viscous flow of electrically conducting fluid is applicable in many industrial processes, such as in magnetic materials processing, purification of crude oil, MHD electrical power generation, glass manufacturing, geophysics, and paper production, etc. Soundalgekar and Murty [11] studied the effect of a magnetic field on the flow of a viscous fluid over a semi-infinite porous plate with suction and injection. Radiation effect on the MHD flow past an isothermal vertical plate was investigated by Chandrakala and Raj [12]. The analytical solution of MHD flow over a vertical plate with constant mass diffusion was obtained by Das [13]. MHD flow past an oscillating vertical plate with Newtonian heating was considered by Hussanan et al. [14]. In another paper, Hussanan et al. [15] extended their own work by considering the mass transfer effect. Ibrahim and Shanker [16] studied MHD flow of a nanofluid over non-isothermal stretching sheet. Bhattacharyya and Layek [17] extended the work initiated by Ibrahim and Shanker [16] to exponentially permeable stretching sheet. Recently, MHD flow and heat transfer of nanofluid over nonlinearly stretching/shrinking sheet was investigated by Pal and Mandal [18]. Khan et al. [19] investigated numerically the heat transfer characteristics of water functionalized carbon nanotube flow over a static/moving wedge. Later on, Haq et al. [20] extended the problem of Khan et al. [19] by considering metallic nanoparticles instead of carbon nanotube. A few other important investigations of MHD flow of a nanofluid have been made recently [21-25].

On the other hand, the problem of MHD flow past an accelerated vertical plate has many practical applications such as filtration process, the drying of porous materials in textile industries and saturation of porous materials by chemicals [26]. In this paper, our main objective is to analyze MHD flow of a nanofluid over an accelerating infinite vertical plate through a porous medium in the presence of thermal radiation. Water as conventional base fluid containing three different types of nanoparticles, namely copper (Cu), alumina (Al₂O₃), titanium dioxide (TiO₂) are considered. Thermo physical properties of base fluid and nanoparticles are given in Table 1. The flow induced by simultaneous action of buoyancy forces and due to accelerating plate. Using suitable transformations, governing equations have been reduced to a set of linear ordinary differential equations. The resulting system has been solved analytically using the Laplace Transform method and presented in closed form.

### Table 1 Thermo physical properties of base fluid and nanoparticles

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>ρ (kg/m³)</th>
<th>cₚ (J/kg K)</th>
<th>k (W/m K)</th>
<th>β×10⁶ (K⁻¹)</th>
<th>φ</th>
<th>σ (S/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water/base fluid</td>
<td>997.1</td>
<td>04179</td>
<td>0.613</td>
<td>21</td>
<td>0</td>
<td>5×10⁻⁶</td>
</tr>
<tr>
<td>Copper (Cu)</td>
<td>8933</td>
<td>385</td>
<td>401</td>
<td>1.67</td>
<td>0.05</td>
<td>59.6×10⁶</td>
</tr>
<tr>
<td>Aluminum Oxide (Al₂O₃)</td>
<td>3970</td>
<td>765</td>
<td>40</td>
<td>0.85</td>
<td>0.15</td>
<td>35×10⁶</td>
</tr>
<tr>
<td>Titanium Oxide (TiO₂)</td>
<td>4250</td>
<td>6.862</td>
<td>8.9538</td>
<td>0.90</td>
<td>0.2</td>
<td>2.6×10⁶</td>
</tr>
</tbody>
</table>

### 2.0 MATHEMATICAL FORMULATION

We consider the unsteady boundary layer flow and heat transfer of a nanofluid through a porous medium over an accelerating vertical plate. The flow is induced by buoyancy forces and due to accelerating plate. The x-axis is directed along the plate, y is the coordinate measured normal to it and the flow being confined to y > 0. It is assumed that the fluid is electrically conducting and the magnetic field is applied perpendicular to the plate. The magnetic Reynolds number is considered to be small enough to neglect the induced magnetic field. It is also assumed that at the initial moment t = 0, both the plate and fluid are at rest with the constant temperature T∞. At time t = 0⁺, the plate begins to accelerate in its own plane with velocity Arⁿ. Physical model and coordinate system are shown in Figure 1.
Under these conditions, the flow is governed by the following set of partial differential equations

\[
\frac{\partial u}{\partial t} = \frac{\mu_f}{\rho_f} \frac{\partial^2 u}{\partial y^2} - \left( \sigma_{nf} B_0^2 + \frac{\mu_{nf} \phi}{k} \right) u + g(\rho \beta)_{nf} (T - T_w), \tag{1}
\]

\[
(\rho c_p)_{nf} \frac{\partial T}{\partial t} = k_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_{nf}}{\partial y}. \tag{2}
\]

The appropriate initial and boundary conditions are given as

\[
u(y,0) = 0, T(y,0) = T_w, \text{ for all } y \geq 0, \tag{3}
\]

\[
u(0,t) = A \mu, T(0,t) = T_w, t > 0, \tag{4}
\]

\[
u(\ast, t) \rightarrow 0, T(\ast, t) \rightarrow T_w, t > 0, \tag{5}
\]

where \(u\) is the velocity, \(B_0\) is the magnetic field, \(\phi\) is porosity of the medium, \(k\) is the permeability, \(A\) and \(n > 0\) are constants, and \(\rho_{nf}, \mu_{nf}, \sigma_{nf}, k_{nf}, \beta_{nf}, (\rho c_p)_{nf}\) are density, dynamic viscosity, electrical conductivity, thermal conductivity, thermal expansion coefficient, heat capacitance of the nanofluid, respectively, which are defined as

\[
\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^2}, \rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s, \nonumber
\]

\[
(\rho c_p)_{nf} = (1 - \varphi)(\rho c_p)_f + \varphi (\rho c_p)_s, \nonumber
\]

\[
\sigma_{nf} = \sigma_f \left[ 1 + \frac{3(\sigma - 1)\varphi}{(\sigma + 2) - (\sigma - 1)\varphi} \right], \quad \sigma = \frac{\sigma_f}{\sigma_s}. \nonumber
\]

Introduce the following non-dimensional variables

\[
y^* = \frac{u}{v_f} y, \quad t^* = \frac{u^2}{v_f^2} t, \quad u^* = \frac{u}{u^*}, \quad \theta = \frac{T - T_w}{T_w}. \tag{6}
\]

Using the Rosseland approximation [15], in the energy equation (2), and implementing equation (6) into equations (2) and (3), we get (* symbols are dropped for simplicity)

\[
\frac{\partial u}{\partial t} = a_1 \frac{\partial^2 u}{\partial y^2} \left( M^2 a_2 + 1 \right) u + Gra_3 \theta, \tag{7}
\]

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr a_4} (a_5 + R) \frac{\partial^2 \theta}{\partial y^2}. \tag{8}
\]

The corresponding initial and boundary conditions are

\[
u(y,0) = 0, \quad \theta(y,0) = 0, \text{ for all } y \geq 0, \tag{9}
\]

\[
u(0,t) = A \mu, \quad \theta(0,t) = 1, t > 0, \tag{10}
\]

\[
\nu(\ast, t) \rightarrow 0, \quad \theta(\ast, t) \rightarrow 0, \quad t > 0, \tag{11}
\]

where

\[
Gr = \frac{v_f \beta_f T_w^3}{U^3}, \quad M^2 = \frac{v_f \sigma_f B_0^2}{\rho_f U^2}, \quad Pr = \frac{a_f c_p}{k_f}, \nonumber
\]

\[
R = \frac{16 \alpha^3 T_w^3}{3k_f} \frac{1}{k}, \nonumber
\]

are the Grashof number, magnetic parameter, Prandtl number, radiation parameter and porosity parameter.

### 3.0 Method of Solution

Applying the Laplace transforms to equations (7) and (8), and using conditions (10-11), we get the following solutions in the transformed \((\nu, \theta)\) plane

\[
\tilde{\nu}(\nu, \theta) = \left( \frac{\Gamma(m + 1)}{q^{(m + 1)}} \right) e^{-\nu \sqrt{q + a_2}} a_1 + \frac{a_5 Gr}{a_1 a_8 q(q - a_8)} e^{-\nu \sqrt{q + a_2}} a_1 \nonumber
\]

\[
- \frac{a_5 Gr}{a_1 a_8 q(q - a_8)} e^{-\nu \sqrt{q}} a_6, \tag{12}
\]

\[
\tilde{\theta}(\nu, \theta) = \frac{1}{q} \left[ e^{-\nu \sqrt{q}} a_6 \right], \tag{13}
\]

which upon inverse Laplace transform results

\[
\theta(\nu, t) = e^{\left( \frac{a_5}{2(a_2 + a_2 \sqrt{q})} \right) \text{erfc} \left( \frac{\nu}{2 \sqrt{t}} - \frac{a_2}{2} \right) - \text{erfc} \left( \frac{\nu}{2 \sqrt{t}} \right) \frac{Pr}{2}}, \tag{14}
\]

\[
u(\ast, t) = \frac{\Gamma(m + 1)}{2\Gamma(m)} \int_0^t (t - s)^{m - 1} e^{-\frac{a_5}{a_1} s} \text{erfc} \left( \frac{\nu}{2a_2 s} - \frac{a_2 a_5}{2a_2 s} \right) ds + \frac{\Gamma(m + 1)}{2\Gamma(m)} \int_0^t (t - s)^{m - 1} e^{-\frac{a_5}{a_1} s} \text{erfc} \left( \frac{\nu}{2a_2 s} + \frac{a_2 a_5}{2a_2 s} \right) ds \tag{15}
\]
\[ u(y,t) = \frac{\Gamma(m+1)}{2\Gamma(m)} \int_0^t (t-s)^{m-1} \left[ -\frac{y}{a_2} e^{-\frac{\sqrt{y-a_7}}{2a_5}} + \frac{\sqrt{y-a_7}}{2a_5} \right] ds. \] (16)

Furthermore, the mechanical part of velocity (16) is also equivalent to the corresponding solution obtained by Ali et al. [27], see equation (13).

### 4.0 Graphical Results and Discussion

In the previous section, analytical solutions are obtained for the velocity and temperature of water based nanofluid past an accelerating infinite vertical plate in a porous medium with constant wall temperature. These solutions depend on several dimensionless parameters such as Grashof number, magnetic parameter, Prandtl number, radiation parameter and porosity parameter. The influence of these pertinent parameters on velocity and temperature is graphically underlined. Geometry of the problem is shown in Figure 1. Figure 2 shows the comparison of different types of nanofluids. It is found that velocity profiles for Al\textsubscript{2}O\textsubscript{3}-water are greater than those of TiO\textsubscript{2}-water and Cu-water nanofluids. This is because the density of Al\textsubscript{2}O\textsubscript{3}-water (3970 kg/m\textsuperscript{3}) has the minimum values as compare to TiO\textsubscript{2}-water (4250 kg/m\textsuperscript{3}) and Cu-water (8933 kg/m\textsuperscript{3}) nanofluids. Figure 3 describes the behavior of the temperature distributions for different types of nanofluids, namely Cu-water, Al\textsubscript{2}O\textsubscript{3}-water and TiO\textsubscript{2}-water, when the other parameters are fixed. From this figure, it is found that the temperature of Cu-water is higher than those of Al\textsubscript{2}O\textsubscript{3}-water and TiO\textsubscript{2}-water nanofluids. The thermal conductivity of Cu-water (401 W/m K) has significantly higher than that of Al\textsubscript{2}O\textsubscript{3}-water (40 W/m K) and TiO\textsubscript{2}-water (8.9538 W/m K). This means that copper's high thermal conductivity allows heat transfer more quickly. In addition, it is noted that the lowest heat transfer rate is obtained for the TiO\textsubscript{2} nanoparticles due to domination of conduction mode of heat transfer.

![Figure 2 Comparison of velocity profiles for different types of nanofluids](image-url)
Further, it is observed that the temperature decreases with the increase of the Prandtl number Pr. This agrees with the physical behavior, when the values of Pr increases the thermal conductivity decreases, and finally, temperature decreases because of low thermal conductivity. On the other hand, temperature increases with increase in radiation parameter R, but the temperature of Cu-water is much higher than the Al2O3-water and TiO2-water nanofluids. This behavior is shown in Figure 4.

5.0 CONCLUSION

This study develops the exact solutions for MHD flow of water based nanofluids past an accelerating infinite vertical plate in a porous medium. Graphs are plotted for embedded parameters and discussed. Results showed that temperature decreases significantly with increasing Prandtl number but it increases when radiation parameter is increased. The results also indicate that there is a significant difference between the Al2O3-water, TiO2-water and Cu-water nanofluids on the temperature. It is found that Cu-water nanofluids prove higher heat transfer performance than Al2O3-water and TiO2-water. Further, the analytical solutions obtained in the present study can be used to verify the validity of obtained numerical solutions for more complicated heat transfer flow problems.

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References


