Thermal Radiation Effect on Hydromagnetic Flow of Dusty Fluid over a Stretching Vertical Surface

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1.0 INTRODUCTION

The study of fluid dynamics due to a stretching surface is essential due to its various applications to engineering and industrial disciplines. These applications include the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning and drawing plastic films. The quality of the resulting sheeting material depends considerably on the flow properties of the ambient fluid, speed of the collection and the rate of heat transfer at the stretching surface. Also the analysis of heat transfer with radiation is important in electrical power generation, solar power technology and other industrial fields. The properties of final product also depend highly on the rate of cooling. Therefore, the study of hydrodynamics flow also becomes important due to its applications in designing cooling systems with liquid metals and others materials processing. In view of these applications, Crane, studied the two-dimensional boundary layer flow due to a stretching sheet. Then, several investigations have been done related to Crane, flow problem with different physical situations have been reported. In the above investigations, they only deal the fluid flow induced by a linear stretching sheet without fluid particle suspension. The study of the boundary layer of fluid particle suspension is significant in determining the particle accumulation and impingement of the particle on the surface. Therefore, Chakrabarti, investigated the boundary layer in a dusty gas. Then, Vajravelu and Nayfeh, studied the boundary layer in a dusty fluid with the presence of magnetic flow. Recently, Gireesha et al. studied the boundary layer flow and heat transfer of a dusty fluid over a stretching vertical surface.

Motivated by these analyses, this paper investigated the effect of thermal radiation and hydrodynamic flow on the dusty fluid over a stretching vertical surface. The coupled nonlinear partial differential equations governing the problem are transformed into a couple nonlinear ordinary differential equations by using similarity transformation. These nonlinear ordinary differential equations are solved numerically by using Runge Kutta Fehlberg forth-fifth order method (RKF45 Method) for different values of parameters of interest such as fluid particle interaction parameter, magnetic parameter, radiation parameter, Grashof number, Eckert number and Prandtl number on the flow.

In this paper, an analysis has been carried out to investigate the hydromagnetic fluid flow of dusty fluid with thermal radiation at vertical stretching sheet. The behavior of velocity and temperature profile of hydromagnetic fluid flow with fluid particle suspension is analyzed by using Runge Kutta Fehlberg forth-fifth order method (RKF45 Method). These solutions are presented and discussed for different parameters of interest such as fluid particle interaction parameter, the magnetic parameter, the radiation parameter, Grashof number, Eckert number and Prandtl number on the flow.

Keywords: Dusty fluid; two phase; magnetohydrodynamic; radiation; heat transfer

Abstract

In this paper, an analysis has been carried out to investigate the hydromagnetic fluid flow of dusty fluid with thermal radiation at vertical stretching sheet. The behavior of velocity and temperature profile of hydromagnetic fluid flow with fluid particle suspension is analyzed by using Runge Kutta Fehlberg forth-fifth order method (RKF45 Method). These solutions are presented and discussed for different parameters of interest such as fluid particle interaction parameter, the magnetic parameter, the radiation parameter, Grashof number, Eckert number and Prandtl number on the flow.

Keywords: Dusty fluid; two phase; magnetohydrodynamic; radiation; heat transfer

Abstrak


Kata kunci: Bendalir berdebu; dua aliran fasa; magnetohidrodinamik; radiasi; pemindahan haba
\[ q_x = -\frac{4\sigma^4}{3k^4} \frac{\partial T^4}{\partial y} \]  

where \( \sigma^4 \) and \( k^4 \) are Stefan-Boltzmann constant and the mean absorption co-efficient, respectively. Assuming that temperature difference within the flow is such that \( T^4 \) may be expanded in Taylor’s series. Expanding \( T^4 \) about \( T_c \) and neglecting higher order will obtain

\[ T^4 \approx 4T_c^3T - 3T_c^4 \]  

Substituting Equation (8) and Equation (9) in Equation (6) reduces to

\[ \frac{u}{\partial x} + \frac{v}{\partial y} = \frac{k}{\partial x} \frac{\partial^2 T}{\partial y^2} + \frac{N}{\rho \tau_x} (T_p - T) \]

\[ \frac{N}{\rho \tau_x} \frac{\partial (u_r - u)}{\partial y} \]

The mathematical analysis of the problem is simplified by introducing the following dimensionless coordinates in term of similarity variable and similarity function, \( \tau \) as

\[ u = bx \tau \eta, \hspace{1em} v = \sqrt{\mu b} f (\eta), \hspace{1em} \theta (\eta) = \frac{T - T_c}{T - T_c} \]

\[ \eta = \frac{b}{\sqrt{\mu b}} y, \hspace{1em} \rho_r = H (\eta), \hspace{1em} u_p = bx F (\eta), \hspace{1em} v_p = \sqrt{\mu b} G (\eta) \]

\[ \theta_r (\eta) = \frac{T_p - T}{T_c - T_p}, \hspace{1em} T - T_c = A \left( \frac{1}{T} \right)^{\theta_r (\eta)} \]

By using the similarity equations from Equation (12), we obtain the following nonlinear ordinary differential equations:

\[ f'' (\eta) + 4f (\eta)f' (\eta) - 2f'' (\eta) = \frac{Gr \theta_r (\eta)}{Pr \tau_x} \]

\[ 1 + \frac{4}{3} R \theta_r (\eta) + Pr f (\eta) = f'' (\eta) - 2f' (\eta) \theta_r (\eta) \]

\[ \left[ 1 + \frac{2}{3} R \right] \theta_r (\eta) + \frac{Pr \tau_x}{\rho \tau_x} \left[ \theta_r (\eta) - \theta (\eta) \right] \]

\[ f (\eta) + H (\eta) \theta_r (\eta) + H (\eta) G (\eta) = 0 \]

\[ G (\eta) F (\eta) + \left[ F (\eta) \right]^2 + 2 \beta \left[ F (\eta) - f' (\eta) \right] = 0 \]

\[ G (\eta) G' (\eta) + \theta_r (\eta) + G (\eta) \theta' (\eta) = 0 \]

\[ 2F (\eta) \theta (\eta) + G (\eta) \theta (\eta) - \frac{c_{e_m}}{c_{e_m} \tau_x} \theta (\eta) = 0 \]

\[ \tau = mk \] is the relaxation time of the particle phase, \( \beta = 1/b \) is the fluid particle interaction parameter, \( Gr = \frac{gb^4 T_c}{2b^2 \Delta x} \) is the Grashof number, \( M = \frac{\sigma_0 b^2}{\rho b} \) is magnetic parameter and \( \rho_r = \rho_r / \rho \) is relative density. While, \( Pr = \mu c_p / k \) is the Prandtl number, \( Ec = c_{e_m} / A \tau_x \) is the Eckert number and \( R = 4\sigma_x T_c^2 / k k^2 \) is the radiation parameter.

The boundary condition in Equation (8) becomes

\[ f (\eta) = 0, \hspace{1em} f' (\eta) = 1, \hspace{1em} \theta (\eta) = 1 \hspace{1em} \theta (\eta) = 0 \]

\[ f (\eta) \rightarrow 0, \hspace{1em} F (\eta) \rightarrow 0, \hspace{1em} G (\eta) \rightarrow -f (\eta) \]

\[ H (\eta) \rightarrow \omega, \hspace{1em} \theta (\eta) \rightarrow 0, \hspace{1em} \theta (\eta) \rightarrow 0 \hspace{1em} \eta \rightarrow \infty \]
3.0 RESULTS AND DISCUSSION

The system of coupled nonlinear ordinary differential equations as in Equations (13) to (18) with boundary condition Equation (19) is solved by using Runge-Kutta Fehlberg forth-fifth order method. The symbolic algebra software Maple is adopted given by Aziz\textsuperscript{11} to solve these equations. Numerical solutions have been carried out to study the effect of various physical parameter such as fluid particle interaction parameter $\beta$, the magnetic parameter $M$, the radiation parameter $R$, Grashof number $Gr$, Eckert number $Ec$ and Prandtl number $Pr$ are shown graphically. In order to verify the accuracy of this study, the value of wall temperature $\theta'(0)$ gradient for different value of Prandtl number are given in Table 1. It shows the excellent agreement with reported by Grubka and Bobba\textsuperscript{4} and Abel and Mahesha\textsuperscript{5}.

<table>
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Figure 1 shows the effect of magnetic parameter on the velocity profile for $\beta=N=Gr=M=R=0$. From this graph it is shown that the velocity of fluid and dust phase decreases as $M$ increases. As $M$ increase the Lorentz force which opposes the flow also increases and decreases the velocity of the flow. While, from Figure 2 shows the effect of Grashof number on velocity profile. It shows that the velocity of both fluid and dust phase increase as $Gr$ increase. Physically, $Gr > 0$ means heating of the fluid or cooling the boundary surface and $Gr < 0$ means cooling of the fluid or heating boundary surface while $Gr = 0$ corresponds to the absence of free convection current.

The ratio of the two components has a profound effect on the microscopic structure and macroscopic properties of the gel in toluene. Figure 3 shows the effect of radiation parameter on temperature profile. It is observed that the increase in the thermal radiation parameter $R$ produces significant increases in the thickness of the thermal boundary layer fluid. Figure 4 shows that the effect of fluid particle interaction parameter on temperature profile. We observed that temperature of the fluid and dust particle decrease with increase in $\beta$ respectively. Figure 5 illustrates the effect of Eckert number on temperature profile of fluid and dust phase. It is evident from these graphs that increases in $Ec$ increasing the temperature for both fluid and dust phase. Figure 6 is plotted for the temperature profile for different values of Prandtl number. We observed that the increasing number of $Pr$ implies the decreasing temperature of fluid and dust phase.
4.0 CONCLUSION

The hydromagnetic fluid flow and heat transfer of a dusty fluid with thermal radiation due to a vertical stretching sheet has been investigated. The effect of some parameters $M, R, \beta, Gr, Ec$ and $Pr$ controlling the velocity and temperature profiles are shown graphically and discussed briefly. Some of the important findings of our study are listed below:

- The effect of $M$ is to decrease the momentum boundary layer thickness.
- The effect of $R$ is to increase the thermal boundary layer thickness.
- The effect of $Gr$ is to increase the momentum boundary layer.
- $Ec$ increases the thermal boundary layer thickness.
- The boundary layers are highly influenced by the Prandtl number. The effect of $Pr$ is to decrease the thermal boundary layer thickness.

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References