ADAPTIVE SPATIAL MODE OF SPACE-TIME AND SPACE-FREQUENCY OFDM SYSTEM OVER FADING CHANNELS

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Abstract. In this paper we present a 2 transmit 1 receive (1 Tx : 1 Rx) adaptive spatial mode (ASM) of space-time (ST) and space-frequency (SF) orthogonal frequency division multiplexing (OFDM). At low signal to noise ratio (SNR) we employ ST-OFDM and switch to SF-OFDM at a certain SNR threshold. We determine this threshold from the intersection of individual performance curves. Results show a gain of 9 dB (at a bit error rate of 10-3) is achieved by employing adaptive spatial mode compared to a fixed ST-OFDM, almost 6 dB to fixed SF-OFDM, 4 dB to Coded ST-OFDM and 2 dB to a fixed coded SF-OFDM, at a delay spread of 700 ns.

Key words: OFDM, space-time, space-frequency, channel coding and multipath fading channels.


Kata kunci: OFDM, ruang-masa, ruang-frekuensi, pengekodan saluran dan saluran pemudaran berbilang laluan.

1.0 INTRODUCTION

Multicarrier system such as Orthogonal Frequency Division Multiplexing (OFDM) is slowly proving to be a good candidate for the next generation

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wireless system to provide high data rate, high capacity and high mobility. This is due to the fact that it is able to combat intersymbol interference (ISI) with the employment of cyclic prefix, as well as providing a high data rate by splitting it into a number of low rate streams. OFDM is currently available commercially for system with short delay, such as indoor and wireless Local Area Network (WLAN) [1] system and simplex broadcasting system such as European Digital Audio Broadcasting (DAB) and Digital Video Broadcasting (DVB) [2-4]. However, OFDM by itself is still unable to cope with multipath fading, commonly found in outdoor mobile radio system, which makes reliable communications difficult.

In recent years there has been an increasing interest of exploring spatial diversity [5] in conjunction with the OFDM technique. Alamouti [5] proposed a 2 transmit and 1 receive (2 Tx:1 Rx) antenna system of Space-Time Block Code (STBC) with Binary Phase Shift Keying (BPSK) modulation that works well in a narrow flat fading environment where channel is static and non dispersive. [6-9] showed that combining STBC with OFDM produces an improved system that would combat frequency selective fading, especially in Multiple Input and Multiple Output (MIMO) system. Unfortunately, this technique requires that fading to be slow to ensure that channel is constant over several OFDM symbols’ duration [10] (depending on the configuration of the space-time scheme). Later works [10-13] introduced space-frequency in the context of OFDM and MIMO, to fully achieve spatial and frequency diversity that would combat both fast fading and frequency-selective fading for broadband channels. Other works include space-time-frequency OFDM (e.g. [14-15]) which exploit transmit diversity in space, time and frequency in order to achieve higher diversity gain.

Early works in space diversity involves in the constructions of various block orthogonal codes, trellis and turbo codes in conjunction with space diversity scheme [16-23]. However, “large delay spread in frequency selective fading channels destroy the orthogonality of the received signal” [24] in frequency selective fading and makes detection of the system more difficult. The use of space diversity in the context of OFDM transforms frequency selective fading of single carrier system into multiple flat fading channels [24-25] and the effects of large delay spread can be mitigated using OFDM cyclic prefix [26], [27] as mentioned earlier. We could also introduce some guard band in between carriers to avoid inter carrier interference among the many adjacent subcarriers in OFDM system.

In this work, we generate 4 different modes; ST-OFDM, coded ST-OFDM, SF-OFDM and coded SF-OFDM at delay spreads ranging from 500 ns to 1.5µs commonly found in outdoor environments. For the coded schemes we concatenate ST or SF codes with channel coding, i.e. convolutional code of
rate 1/2 with length of 5. We build our adaptive spatial mode (ASM) OFDM based on the performance results of each scheme. The study was motivated from the previous results [8-14] that showed ST-OFDM can perform well only in slow fading environment while SF-OFDM in fast fading channel [28]. Combining this information and making use of our individual (ST-OFDM and SF-OFDM) performance results we determine the switching thresholds between each scheme from the intersections of their performance curves. For the initial stage, we assume that channel state information (CSI) is obtained via a dedicated feedback channel or by channel reciprocity when time division duplex is employed, where uplink and downlink share the same channel [24, 29].

The outline of this paper is as follows. In Section 2 we briefly describe a conventional OFDM. In Section 3 we elaborate ST coding and SF block code in the context of OFDM. Section 4 explains the channel model used in the simulation. Section 5 presents the simulation and parameters setup as well as some significant results and we conclude in Section 6.

**Notation**

Throughout the paper we adopt the following notational conventions in our mathematical expressions. We denote matrix vectors by bold upper case letters, and column or row vectors by bold lower case letters. Superscripts \((.)^T\) and \((.)^*\) are used to denote transpose and conjugate of a matrix.

### 2.0 ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING

Figure 1 shows a simplified diagram of an OFDM system in which a binary input sequence is modulated to QAM symbols with symbol interval, \(T_s\). The serial to parallel converter collects the serial data symbols \(S(n)\) into length \(N\) vector \(S= [S(0) S(1) \ldots S(N-1)]^T\) where \(N\) is the number of subcarriers in the OFDM system. This vector is then modulated by an inverse fast Fourier transform (IFFT) into an OFDM symbol sequence as follows [6]:

\[
s(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S(k)e^{j2\pi nk/N}, \quad 0 \leq n \leq N - 1
\]

A cyclic prefix of length \(P\) of the modulator output is added as a guard interval with resulting sequence, \(s_p\) \([s_p(0)s_p(1) \ldots s_p(N+P-1)]^T\). The sequence is transmitted through a frequency selective fading channel of order, \(L\) defined as [30],

\[
h(n, \tau) = \begin{cases} 
0 & \tau = L, \\
0 & \tau > L. 
\end{cases}
\]
The guard interval $P$ is chosen such that $P \geq L$ to avoid intersymbol interference (ISI). The chosen length ensures that channel has died down before the receipt of the next sequence. Convolving the prefixed sequence $s_p(n)$ with results in a received sequence, as follows:

$$rp(n) = \sum_{k=0}^{L} s_p(n-k) \cdot h(n, k) + w(n), \quad 0 \leq n \leq N + P - 1$$  \hspace{1cm} (3)

where $w(n)$ is the additive white Gaussian noise (awgn) of the channel. Assuming the channel impulse response remains constant during the entire block intervals, the time-varying response $h(n, k)$ can be replaced by the time-invariant response $h(k)$. At the receiver, the cyclic prefix is removed from the received sequence such that we have $r(n) = r_p(n + P)$, $0 \leq n \leq N - 1$. The OFDM demodulator performs an $N$-point fast Fourier transform (FFT) on the sequence to yield the demodulated output $R(n)$. Notice that with $s_p(n)$ constructed as the cyclic extension of $s(n)$, the sequence $r(n)$ is the cyclic convolution of $s(n)$ and $h(n)$ resulting in multiplication in the frequency domain. Therefore, we can express the FFT output as

$$R(n) = H(n) \cdot S(n) + W(n), \quad 0 \leq n \leq N - 1$$  \hspace{1cm} (4)

where $H(n)$ is the FFT output of the channel impulse response, and $W(n)$ is the FFT output of the channel noise. Note also that OFDM with cyclic prefix transforms a frequency selective fading subchannels into $N$ flat fading subchannels [6, 10, 25]. These flat fading subchannels provide a good platform on which space-time or space-frequency or even space-time-frequency processing techniques developed for flat fading channels, can be applied.
3.0 SPATIAL DIVERSITY WITH TIME AND FREQUENCY BLOCK CODING

The simple OFDM model is extended to space-time/space-frequency (ST/SF)-OFDM with $N_t$ transmit and $N_r$ receive antenna as shown in Figure 2.

We concatenate the ST/SF coded symbols with channel coding to further combat channel impairments. Each coded vector $X_i$ is then inverse fast Fourier transformed (IFFT) and cyclic-prefixed. The prefixed signals are then passed through their respective antenna. Each transmitted stream will undergo the influence of multipath fading and additive white Gaussian noise. At the receiver, cyclic prefix is removed and fast Fourier transform (FFT) is performed to recover the received symbols. These symbols can be estimated using maximum likelihood decoding or by a simple zero forcing scheme. The symbols are then demodulated and decoded using Viterbi algorithm and the recovered bits are then compared with the transmitted bits for BER measurement. Throughout our work we assume that channel is static during one OFDM symbol in SF-OFDM and two consecutive OFDM symbols in ST-OFDM systems.

3.1 Coded Space-Time OFDM

A simple ST-OFDM scheme using 2 Tx:1 Rx antenna was first proposed by Lee and Williams [6]. We extend the authors’ work by concatenating this ST block coded OFDM with convolutional codes as channel coding. Consider the successive data symbol vectors in which the $M$-th block is denoted by vector $X_1$ and the $(M+1)$-th block as $X_2$ as follows:
Note that each QAM modulated data symbol \( S(n) \) has been channel coded in Equation (5). Adopting Alamouti’s scheme [5], we transmit \( X_1 \) from transmitter 1 during the first time slot and \(-X_2^*\) in the second time slot. At the second transmitter, we transmit in the first time slot followed by \(-X_1^*\) at time slot 2 resulting in Alamouti’s 2 \( \times \) 2 transmission matrix,

\[
A X X_2 = \begin{bmatrix}
X_1 & X_2 \\
-X_2^* & X_1^*
\end{bmatrix}
\]

(6)

where row and column denotes time and space, respectively. In order for the receiver to detect received signals by means of linear processing, it is important that data symbol vectors of each transmit antenna are orthogonal. Note that orthogonality means that Equation (7) is satisfied such that the elements of the transmission matrix are linear combinations of indeterminate \( X_1, X_2, \ldots, X_k \) and their conjugates [17],

\[
AA^* = (|X_1|^2 + |X_2|^2 + \ldots + |X_k|^2)I
\]

(7)

where \( I \) is the identity matrix.

Now let \( H_1 \) and \( H_2 \) be two diagonal matrices whose diagonal elements are the FFTs of the respective channel impulse response \( h_1 \) and \( h_2 \). \( H_1 \) and \( H_2 \) are diagonal matrices due to the fact that each channel path are assumed to be independent of each other, while each element in \( H_1 \) and \( H_2 \) are decoupled from one another. This is due to the fact the frequency selective fading has been transformed into a number of flat fading channels through OFDM such that each channel element is independent from each other. With this assumption, the demodulated vectors in the corresponding time slots after FFT operation are given by,

\[
Y_1 = H_1 X_1 + H_2 X_2 + W_1
\]
\[
Y_2 = -H_1 X_2^* + H_2 X_1^* + W_2
\]

(8)

Assuming that channel responses are known at the receiver, the decision variables are constructed by combining both \( X \) and \( \hat{X} \) and channel responses matrices as follows,

\[
\hat{X}_1 = H_1 Y_1 + H_2 Y_2^*
\]
\[
\hat{X}_2 = H_1 Y_2 - H_1
\]

(9)
\( \hat{X}_i \) is estimated from Equation (8) by forcing the other symbol, \( \hat{X}_j = 0 \) where \( j \neq i \). This method works well with low-level constellation but some kinds of detection method such as maximum likelihood decoder or sphere decoder need to be employed for higher constellation [31].

Figure 3 shows ST-OFDM block code which is formed from the 2 transmit antennas and \( N \) subcarriers over two successive OFDM symbols.

**3.2 Space-Frequency OFDM**

For SF-OFDM scheme using 2 Tx:1 Rx antenna we arrange the data symbol vectors such that,

\[
X_1 = [S(0) - S(1)*, S(2) - S(3)*, \ldots, S(N - 2) - S(N - 1)*]^T \\
X_2 = [S(1), S(0)*, S(2), \ldots, S(N - 1), S(N - 2)*]^T
\]

(10)

Now let \( X_e \) and \( X_o \) be two length \( N/2 \) vectors denoting the even and odd component of \( X \) as follows,

\[
X_e = [S(0), S(2), \ldots, S(N - 4), S(N - 2)]^T \\
X_o = [S(1), S(3), \ldots, S(N - 3), S(N - 1)]^T
\]

(11)

Similarly if we let \( X_{1,e}, X_{2,o}, X_{1,o} \) and \( X_{2,e} \) be the even and odd component vectors of and , we can express Equation (11) as:

\[
X_{1,e} = X_e, \quad X_{1,o} = -X_o \\
X_{2,e} = X_o, \quad X_{2,o} = X
\]

(12)
The demodulated signal at the receiver is:

\[ X = H_1X_1 + H_2X_2 + W \]  \hspace{1cm} (13)

We can write Equation (13) in terms of even and odd components as follows:

\[ Y_e = H_1,eX_{1,e} + H_2,eX_{2,e} + W_e \]
\[ Y_o = H_1,oX_{1,o} + H_2,oX_{2,o} + W_o \]  \hspace{1cm} (14)

With the assumption that CSI is known at the receiver, SF-OFDM decoder constructs the following estimates before passing it to maximum likelihood detector:

\[ \hat{X}_e = H_1,e Y_e + H_2,o Y_o \]
\[ \hat{X}_o = H_2,e Y_e - H_1,o Y \]  \hspace{1cm} (15)

For 4-QAM constellation, the detection of symbol is made easy using the signum function from Matlab after separating the real and imaginary parts of the estimated symbols. Figure 4 shows SF-OFDM forms space-frequency block codes over 2 transmit antennas and \( N \) subcarriers occupying one OFDM symbol.

**Figure 4** A 2-dimensional diagram showing SF coding across an OFDM symbol \([N_s]\), with number of transmit antenna \(N_t=2\), and \(N\) subcarriers.
3.3 Code Rate

Consider that X has been generated by $N_M$ QAM symbols in the block vector $S = [S(0) \ S(1) \ ... \ S(N_M - 1)]^T$. Transmitting $N_M$ symbols using $N$ subcarriers occupying $N_S$ OFDM symbols, yield a code rate of:

$$R_{ST/SF-OFDM} = \frac{N_M}{N_S N} \text{ bps/Hz.} \quad (16)$$

Accounting for CP with length $P$ and constellation size, $B$ ($4$-QAM corresponding to $2$ bits/symbol), our transmission rate becomes,

$$R_{ST/SF-OFDM} = \frac{N_M}{N_S(N + P)\log_2 B} \text{ bps/Hz.} \quad (17)$$

Note that this code rate does not take channel coding rate into account.

4.0 CHANNEL MODEL

To measure the delay and fading caused by multipath, we choose Naftali channel model [32] of IEEE 802.11 WLAN system. Using this model we compose the channel impulse response of complex samples using random uniformly distributed phase and Rayleigh distributed magnitude. According to [32, 33] the Naftali channel model can be described by,

$$h_k = N\left(0, \frac{\sigma_k^2}{2}\right) + jN\left(0, \frac{\sigma_k^2}{2}\right) \quad (18)$$

where,

$$\sigma_k^2 = \sigma_0^2 \exp(-kT_s/T_{rms})$$
$$\sigma_0^2 = 1 - \exp(-T_s/T_{rms}) \quad (19)$$

with $k$ taking on the index of sample $1$ to maximum number of samples $N\left(0, \frac{\sigma_k^2}{2}\right)$; taking on zero mean Gaussian random variable with variance $\frac{\sigma_k^2}{2}$. 
which is produced by generating an $N(0,1)$ Gaussian random number and multiplying it by $\frac{\sigma_k}{\sqrt{2}}$; $\sigma_k^2 = 1 - \exp(-T_s/\tau_{rms})$ is chosen so that the condition $\sum_{k=0}^{k_{\text{max}}} \sigma_k^2 = 1$ is satisfied to ensure the same average received power. $T_s$ represents the sampling period, and $\tau_{rms}$ represents the delay spread of the channel. The number of samples to be taken in the impulse response should ensure sufficient decay of impulse response tail so that to avoid inter symbol interference. For a sampling rate of 11 MHz, we have $T_s = 1/f_{\text{sample}} = 91$ ns. For a delay spread of $\tau_{rms} = 200$ ns, we have $k_{\text{max}} = 10200/90=23$. The corresponding channel model with the chosen values is shown in Figure 5.

![Figure 5](image-url)
5.0 SYSTEM PARAMETERS SETUP AND SIMULATION RESULTS

Our simulation is performed based on the following system parameters shown in Table 1. Unlike [17] we use larger number of subcarriers to fully exploit the advantage of multicarrier OFDM. Other simulation parameters are as shown in Table 1 with remarks explaining how they are chosen. We measure performance of each scheme, namely ST, coded ST, SF, and coded SF-OFDM at various delay spreads ranging from 500 ns to 1.5 µs delay spread.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating frequency, $f_c$</td>
<td>5.25 GHz</td>
<td></td>
</tr>
<tr>
<td>Number of subcarriers, $N$</td>
<td>1024</td>
<td>$N \times \Delta f \leq BW$</td>
</tr>
<tr>
<td>Delay spread</td>
<td>500 ns to 1.5 µs</td>
<td>Very high data rate</td>
</tr>
<tr>
<td>Bit rate</td>
<td>200 Mbps</td>
<td>Broadband</td>
</tr>
<tr>
<td>Broadband bandwidth, $BW$</td>
<td>200 MHz</td>
<td>$BW/N$</td>
</tr>
<tr>
<td>OFDM subcarrier spacing, $\Delta f$</td>
<td>195.3125 kHz</td>
<td>$1/\Delta f$</td>
</tr>
<tr>
<td>OFDM symbol duration, $T_{OFDM}$</td>
<td>5.12 µs</td>
<td>At least 25% of $T_{OFDM}$</td>
</tr>
<tr>
<td>Cyclic prefix, $T_{CP}$</td>
<td>1.28 µs</td>
<td></td>
</tr>
<tr>
<td>Constellation</td>
<td>4-QAM</td>
<td></td>
</tr>
<tr>
<td>FFT points</td>
<td>1024</td>
<td>Higher number of FFT-point may be used for oversampling purposes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Switching from ST to SF</td>
</tr>
<tr>
<td>Thresholds</td>
<td>2.0, 3.0, 4.0 dB</td>
<td></td>
</tr>
</tbody>
</table>

Note that to achieve a very high speed data rate of 200 Mbps, we would require 1024 bits per OFDM symbol (200 Mbps $\times$ 5.12 µs). Since we chose 4-QAM constellation of 2 bits/QAM-symbol/subcarrier, we would require at least 512 subcarriers. We may choose higher number of subcarriers for oversampling, folding, or training purposes. Choosing $N = 1024$ and with $\Delta f = 195.3125$ kHz, total bandwidth used is exactly 200 MHz.

5.1 Simulation Results and Discussion

We simulate ST-OFDM, SF-OFDM and their coded schemes and compare their performances in terms of bit error rate (BER) of the received signals. Figures 6 and 7 show the BER plot for ST-OFDM and Coded ST-OFDM against SNR, respectively. We show that concatenating channel coding with
**Figure 6** BER versus SNR of a 2 transmit and 1 receive antenna of a 4-QAM ST-OFDM at various delay spreads

**Figure 7** BER versus SNR of a 2 transmit and 1 receive antenna of a Coded 4-QAM ST-OFDM at various delay spreads
ST codes improves the BER but only for shorter delay spread. Note that for symbol duration of $T_{\text{OFDM}} = 5.12\mu s$, the maximum tolerable spread is $1.28\mu s \ (25\% \ of \ T_{\text{OFDM}})$, thus we would expect our system to deteriorate at delay spread greater than $1.28\mu s$ and may not be corrected using channel coding as shown in Figure 8.

![Figure 8 BER versus SNR of a 2 transmit and 1 receive antenna of a 4-QAM SF-OFDM at various delay spreads](image.png)

Unlike ST-OFDM, the performance of SF-OFDM and coded SF-OFDM (Figures 8 and 9, respectively) improves when compared to their ST counterparts. BER is further improved when channel coding is employed, which is translated into coding gain. Diversity gain (the slope of the plot) is the same for both ST and SF-OFDM systems, which is equal to 2.

To clearly compare the overall performance of the three schemes, we plot BER against SNR at 2 extreme delay spreads of 700 ns and 1.0 $\mu s$, shown in Figures 10 and 11, respectively. Coded SF-OFDM outperforms the other schemes in both conditions at almost all given SNRs. Note that to achieve a BER of 10-3 at delay spread of 700 ns, Coded SF-OFDM requires only 8 dB of SNR, a gain of about 3 dB compared to coded SF-OFDM and a gain of 7 dB to ST-OFDM.
5.2 Adaptive Spatial Mode

Our adaptive spatial mode is an adaptive transmission where a mode of either ST or SF is chosen based on the SNR threshold. We determine the threshold from the intersection of individual performance curve of each scheme. In order to select the optimum threshold value, we simulate adaptive spatial mode at 3 different values, ranging from 2.0 dB to 4.0 dB switching from ST-OFDM to
SF-OFDM at a delay spread of 700 ns. We transmit ST-OFDM at lower SNR and move to coded SF-OFDM at higher SNR. Result of BER versus SNR for different value of thresholds is shown in Figure 12. We found a threshold of 3.0 dB to be the optimum value and use this value for comparison purposes against the other schemes. Adaptive spatial mode OFDM outperforms its closest counterpart Coded SF-OFDM by 2 dB at BER of $10^{-3}$ and about 4 dB to coded ST-OFDM, as shown in Figure 13.
6.0 CONCLUSIONS

We have presented an adaptive spatial mode consisting of ST-OFDM and SF-OFDM transmissions. Results show that our proposed adaptive scheme outperforms other schemes by as much as 9 dB (at a bit error rate of $10^{-4}$) compared to a fixed ST-OFDM, about 6 dB to fixed SF-OFDM, 4 dB to Coded ST-OFDM and 2 dB to a fixed coded SF-OFDM, at a delay spread of 700 ns.

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