Prediction of Unmeasured Mode Shape Using Artificial Neural Network for Damage Detection

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Abstract

Artificial neural networks (ANNs) have received much attention in the field of vibration-based damage detection since the 1990s, due to their capability to predict damage from modal data. However, the accuracy of this method is highly dependent on the number of measurement points, especially when the mode shape is used as an indicator for damage detection. With a high number of measurement points, more information can be fed to the ANN to detect damage; therefore, more reliable results can be obtained. Nevertheless, in practice, it is uneconomical to install sensors on every part of a structure; thus the capability of ANNs to detect damage is quite limited. In this study, an ANN is applied to predict the unmeasured mode shape data based on a limited number of measured data. To demonstrate the accuracy of the proposed method, the results are compared with the Cubic Spline interpolation (CS) method. A parametric study is also conducted to investigate the sensitivity of the number of measurement points to the proposed method. The results show that the ANN provides more reliable results compared to the CS method as it is able to predict the magnitude of mode shapes at the unmeasured points with a limited number of measurement points. The application of a two-stage ANN showed results with a high potential for overcoming the issue of using a limited number of sensors in structural health monitoring.

Keywords: Artificial neural network; cubic spline; mode shape; damage detection

Abstrak


Kata kunci: Artificial neural network; cubic spline; mode shape; damage detection

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1.0 INTRODUCTION

Vibration-based damage detection has been investigated by many researchers since the 1990s. An extensive overview of the vibration-based methods can be found in Doebling et al. [1] and Sohn et al. [2]. The accuracy of vibration-based damage detection is very dependent on the number of measurement points, whereby a high number of measurement points will provide better accuracy...
in detecting damage in a structure. As damage can affect the structural performance, it is crucial to access the structure’s condition in as much detail as possible. A large structure means that the size of the response must be recorded from a large number of locations. However, in practice it is rather difficult to obtain measurements at every point on a structure, especially for structures with a large degree of freedom. For this reason, various methods have been proposed to overcome the problem of a limited number of measurement points such as reduction and expansion techniques [3–6], substructuring techniques [7,8], model updating [9,10], the multi-stage assessment scheme [11], and the conventional cubic spline method.

Among those techniques, the Cubic Spline interpolation (CS) method has gained attention in many studies due to its ability to interpolate mode shape values based on a limited number of measurement points. Parloo et al. [12] applied the CS method to interpolate the mode shape values of an aluminum beam structure in several cases of crack formations. Nine measurements are obtained in each data set along the full length of a 480 mm beam, and the CS method is used to calculate the required curvature values of mode shape estimates. Meo and Zumpano [13] utilised the CS method to interpolate the mode shape values of six different optimal sensor-placement techniques and compared them with the numerical model. Hadjileontiadis and Douka [14] used the CS method to predict the missing data in a fractal dimension analysis to detect the existence of cracks in steel plates, while Ooijevaar et al. [15] derived a modal strain energy damage index algorithm using the CS method to detect damage in a delaminated composite T-beam. Other studies employing the CS method to determine the unmeasured mode shape values can be found in Loutridis et al. [16], Rucka and Wilde [17], Bayissa et al. [18] and Radzienksi et al. [19]. Although many researchers concluded that the CS method is capable of interpolating the mode shape values from a limited number of data, it is still unable to provide high-accuracy interpolation. This is because the method is quite sensitive to the sensor arrangement and the number of points measured. Moreover, the method is unable to provide an accurate result if the number of control points is insufficient and the location points selected are inappropriate.

Over the last few decades, there have been many publications on the capabilities of Artificial Neural Networks (ANNs) in the field of vibration-based damage detection. Early works which applied ANNs in the field were done by Rhim and Lee [20], Pandey and Barai [21], Masri et al. [22], and Zhao et al. [23]. In a recent study by Gonzalez-Perez and Valdes-Gonzalez [24], an ANN is utilised to predict damage in the girders of an analytical vehicular bridge. Wang and He [25] demonstrated that the reduced natural frequencies in an arch dam can be detected using an ANN. A numerical simulation and a model experiment were employed in the study. All of the studies concluded that ANNs are robust and capable of detecting relatively small changes in the structural parameters. However, in previous studies, the inputs into the ANN models generally require a complete set of measurement points for a robust result. A complete set of measurements may lead to a larger size of training data for ANN models. If the number of measurement points is insufficient, the accuracy of the result is jeopardised. Among the previous studies that have used ANNs for damage detection, there are no publications known to the authors in which the unmeasured points between the measurements were considered.

In this paper, an approach using an ANN to predict the values of mode shapes at unmeasured points, followed by a damage identification process, is demonstrated. The study is presented in two stages; in the first stage, the ANN is utilised to predict the mode shape values at unmeasured points, while the second stage deals with damage detection. In the first stage, the ANN is trained to relate the measured frequency and mode shape with the unmeasured mode shape values. Once trained, the ANN is then used to predict the mode shape values based on the measured modal data. The accuracy of the predicted mode shapes is then compared with the interpolated mode shape values by the CS method. McKinley and Levine [26] provided the detail of the CS method. A sensitivity study on the effect of the number of measurement points and their locations on the prediction results is also conducted. In the second stage, the predicted mode shape values from the first-stage ANN together with the existing measured modal data are used as the input to the ANN to predict damage locations and their severities. A detailed parametric study is carried out to investigate the feasibility of the proposed method in damage detection. A two-span reinforced concrete slab is used as an example in this study.

### 2.0 METHODOLOGY

#### 2.1 ANN Architecture

The architecture of the ANN used in this study is briefly discussed in this section. A feedforward backpropagation ANN with one hidden layer is used in the study. The tangent sigmoid transfer function is applied to the input and hidden layers while a linear transfer function is applied to the output layer. The ANN models are trained using the Scaled Conjugate Gradient algorithm with an early stopping method. The optimal number of hidden neurons in this study has been determined by a trial and error method. All ANN models are developed on the Matlab platform.

As mentioned in the previous section, there are two stages in the study, involving two ANN models, ANN1 and ANN2, as shown in Figure 1. ANN1 is used to obtain the mode shape values of the unmeasured locations based on measured modal data. The input parameters are the first three mode shapes and natural frequencies of the structure. The training data are generated randomly from the finite element model of a two-span concrete slab, which is discussed in Section 3. The outputs of ANN1 are the predicted mode shape values at the unmeasured points. In the second stage of the study, the inputs to the ANN models are the measured modal data together with the outputs of the ANN1 model (refer to Figure 1). The outputs of ANN2 are the Stiffness Reduction Factor (SRF). SRF indicates the change in the stiffness parameter or the damage severity for each element. The higher the SRF, the more severe is the damage [27]. SRF is denoted as:

$$ SRF = 1 - \frac{E_{\text{damaged}}}{E_{\text{undamaged}}} \; ; \; E = \text{Young Modulus}. $$

![Figure 1 Architecture of the two-stage ANN models](image-url)

This study also examines in greater detail the sensitivity of the proposed two-stage ANN for damage detection. In the sensitivity study, the results from the two-stage ANN are compared to those from the conventional ANN. The conventional ANN consists of only one layer of input, the hidden layer, and the output layer. Figure 2 presents the architecture of the conventional...
ANN. The only inputs to the conventional ANN models are the first three mode shapes of all limited measurement points and the first three natural frequencies of the slab, while the outputs from the ANN are also the SRF values of each element. The rest of the parameters in the study remain the same, for consistency of comparison.

**Figure 2** Architecture of the conventional ANN

### 3.0 NUMERICAL EXAMPLE

A continuous two-span reinforced concrete slab is used as a numerical example to demonstrate the proposed two-stage ANN for damage detection. The slab is modelled using Structural Dynamics Tools (SDT), which runs on the Matlab platform. The dimensions of the slab are 6.4 m × 0.8 m × 0.1 m and it is simply supported at 0.2 m, 3.2 m, and 6.2 m from the left end. “Simply supported” in this study refers to the restraints in all displacements along the global coordinate axes. The material properties of the slab are $E = 3.3 \times 10^{10}$ N/m$^2$, $\rho = 2.45 \times 10^3$ kg/m$^3$, and $\nu = 0.2$. The slab is modelled using shell elements with 165 nodes and 128 elements, and the supports are assumed to be simply supported along nodes 6 to 10, 81 to 85, and 156 to 160. The standard shell elements in the SDT have been applied to all elements. Each of the quadrilateral elements has four integration points for finite element analysis as shown in Figure 3(a). The simulation results are the mode shape values which are measured every 0.2 m on the centreline of the slab model along the span. For the purpose of damage detection, the slab is divided into 32 segments whereby each segment is equally distributed vertically and has dimensions of 0.2 m × 0.8 m, as shown in Figure 3(b). In this example, it is assumed that the mode shapes are measured only at nodes 3, 58, 108, and 163, while the mode shape values at the rest of the nodal points are considered as unmeasured modal data.

To demonstrate the accuracy of the proposed approach in predicting the mode shape values at the unmeasured points, an ANN model is trained to relate the measured modal data with the unmeasured mode shape values. At this stage, only ANN1 is utilised. Table 1 lists the unmeasured nodal points and the measured nodal points.

In this example, the inputs are the first three frequencies and mode shapes measured at the four nodal points while the outputs are the mode shape values at the unmeasured nodal point as listed in Table 1. For training purposes, 3000 training data are used. These training data consist of various damage cases that are generated from the finite element model. The damage cases are simulated by reducing the Young’s modulus ($E$) in the selected segments. The damage cases are varied between the ranges of 0.2 × $E$ and 1.0 × $E$. The training data are divided into sets containing 70%, 20%, and 10% of the data, respectively, for training, validation, and testing purposes. Once trained, two simulated damage cases (Case A and Case B) are fed into the trained ANN1 model to predict the mode shape values at the unmeasured points.

Case A consists of damages in segments 10 and 30 of the slab while Case B consists of damages at segments 4, 5, 12, 20, and 26 with higher severity, as listed in Table 2. Table 3 tabulates the first three frequencies of the simulated damage cases. Figure 4 shows the mode shapes of those two scenarios obtained from the four measurement points.
Table 1 List of measured and unmeasured nodes

<table>
<thead>
<tr>
<th>Measured nodal point</th>
<th>Unmeasured nodal point</th>
</tr>
</thead>
</table>

Table 2 Simulated damage scenarios

<table>
<thead>
<tr>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment</td>
<td>E value</td>
</tr>
<tr>
<td>10</td>
<td>0.6 × E</td>
</tr>
<tr>
<td>30</td>
<td>0.8 × E</td>
</tr>
<tr>
<td>12</td>
<td>0.8 × E</td>
</tr>
<tr>
<td>26</td>
<td>0.6 × E</td>
</tr>
</tbody>
</table>

E = Young Modulus

Table 3 Natural frequencies of the undamaged and the damage cases

<table>
<thead>
<tr>
<th></th>
<th>Undamaged</th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>18.51</td>
<td>18.12</td>
<td>17.77</td>
</tr>
<tr>
<td>Mode 2</td>
<td>29.02</td>
<td>28.73</td>
<td>28.03</td>
</tr>
<tr>
<td>Mode 3</td>
<td>74.04</td>
<td>73.54</td>
<td>69.70</td>
</tr>
</tbody>
</table>

Unit: Hz

Figure 4 Mode shapes of damage Case A and Case B

Figure 5 Comparison of mode shapes in Case A

To assess the accuracy of the proposed approach, the results of ANN1 are compared to finite element mode shapes and mode shapes obtained by the CS method measured at the four measurement points as mentioned above. Figure 5 shows the comparison in Case A. Based on the figure, it is seen that the ANN predictions are well matched to the finite element mode shapes for all modes. For mode 1, the mode shape predicted by the ANN is close to the finite element mode shapes while the CS mode shape is inaccurately interpolated, especially between the first and second measurement points (nodes 3 and 58) and between the third and
fourth measurement points (nodes 108 and 163). In mode 2, it is also observed that the ANN is able to predict the mode shape values in the unmeasured data close to the finite element mode shape, while the CS method provides higher error especially at the middle support. The same situation is observed in mode 3, where the ANN is also able to predict the mode shape values more satisfactorily than the CS method. Another criterion to assess the capability of the proposed approach involves measuring the mean squared error (MSE) between the predicted mode shape and the mode shape calculated by the CS method with the finite element model mode shape. The MSE values are intended to show the precision of the proposed approach with respect to the actual values compared to the CS method, as listed in Table 4.

It is observed that the MSE values of the CS method in both cases are higher than the MSE values of the ANN; this indicates that the proposed method provides a better mode shape estimation compared to the CS method. As mentioned earlier, the reason for the inaccuracy of the CS method is that it is highly influenced by the number and location of measurement points in estimating the unmeasured mode shape values. As for the ANN model, once it is successfully trained, it is able to predict the unmeasured mode shape from the measured modal data without depending on the number and location of the measurement points. A faulty interpretation of inaccurate curves of mode shapes can also be avoided by using the proposed approach. A more detailed study of the sensitivity of the measurement points is conducted in the following section.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>MSE of ANN and CS to actual values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case A</td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>3.19 exp-09</td>
</tr>
<tr>
<td>Mode 2</td>
<td>8.47 exp-09</td>
</tr>
<tr>
<td>Mode 3</td>
<td>8.01 exp-08</td>
</tr>
<tr>
<td>Case B</td>
<td></td>
</tr>
<tr>
<td>Mode 1</td>
<td>1.03 exp-08</td>
</tr>
<tr>
<td>Mode 2</td>
<td>2.27 exp-08</td>
</tr>
<tr>
<td>Mode 3</td>
<td>2.23 exp-07</td>
</tr>
</tbody>
</table>

3.1 Sensitivity Study

3.1.1 Different Numbers of Measurement Points

A more comprehensive study is conducted to investigate the effect of the number of measurement points on the prediction results. For this purpose, the number of measurement points is varied. The prediction results for both the ANN and CS methods are compared with the actual mode shape obtained from the finite element model. The simulated damage case of Case A is used as the testing data, whereby the damages are applied at segments 10 and 30 with 0.6 $\times$ $E$ and 0.8 $\times$ $E$, respectively. Three ANNI models are developed for the purpose of comparison. These ANNI models are distinguished based on different numbers of measurement points used to measure the mode shape. These cases are referred to as 6P, 8P, and 16P for 6, 8, and 16 measurement points, respectively. Figures 6(a) to (d) show the locations of measurement points for the three cases. The corresponding measured and unmeasured nodal points are listed in Table 5.

In each case, it is assumed that the mode shapes are measured at the specified sensor points on the slab. As the first three modes are considered in this study, the inputs to the ANNI models are the first three mode shape values at every measurement point together with the first three modes of natural frequencies. Once trained, the ANNI models are used to predict the mode shape values at the remaining unmeasured points on the slab. The comparisons of the results are illustrated in Figure 7. The known measurement points are denoted by $X$.

Based on the results, it is seen that the ANN is able to predict the mode shape at unmeasured points accurately for all cases. On the other hand, the mode shape curves interpolated by the CS method in Case 6P for modes 2 and 3 are still imperfect compared to the ANN predictions. Higher error is observed in mode 2 at the middle support of the slab for the mode shape interpolated by the CS method. The same scenario is also observed for mode 2 in Case 8P. For Case 16P, both the ANN and the CS method provide good generalisation of the finite element mode shapes. From the results, it is clear that the capability of the CS method improves as the number of measurement points increases while the ANN model is still able to predict the mode shape accurately with a limited number of measurement points. For a further comparison of the capabilities of the two methods, MSE values are calculated between the predicted values and the actual mode shape values. The MSE values for both methods are listed in Table 6. It is obvious that the MSE values obtained for ANNI models are significantly lower than those for the CS method in all cases. This affirms that the proposed approach provides better results compared to the standard CS method. The results also indicate that the proposed approach is able to avoid the need for too many measurement points for mode shape measurement.
Table 5  Complete list of measured and unmeasured nodes.

<table>
<thead>
<tr>
<th>Case</th>
<th>Measured Nodal Point</th>
<th>Unmeasured Nodal Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>8P</td>
<td>3, 23, 43, 63, 103, 123, 143 and 163</td>
<td>8, 13, 18, 23, 28, 38, 43, 48, 53, 63, 68, 73, 78, 83, 88, 93, 98, 103, 113, 118, 123, 128, 138, 143, 148, 153 and 158</td>
</tr>
</tbody>
</table>

(c) Sensor locations for 8 measurement points.

(d) Sensor locations for 16 measurement points.
Table 7 exhibits all the mode shape curves obtained from both the CS method, which are assumed to be the actual mode shapes. Figure 8 compared with the finite element mode shapes of damage in Case method, the mode shapes obtained from both methods are then compared. To assess the effectiveness of the proposed method, the mode shapes obtained from both methods are then compared with the finite element mode shapes of damage in Case A, which are assumed to be the actual mode shapes. Figure 8 clearly shows that the CS method is very dependent on the location of measurement points, while the ANN, once trained with sufficient data, is able to predict the mode shape curves correctly. This proves that the CS method is more sensitive to measurement locations than the ANN method. Therefore, by using the ANN approach, the use of a rigorous process to select the measurement points for damage detection can be avoided.

### 3.1.2 Influence of Location of Measurement Points

This section investigates the influence of different measurement locations on the ANN prediction results in comparison with the CS method. The same simulated damage scenario as Case A is used as the testing data in this section. For this purpose, four cases with different measurement locations are created. Table 7 lists the distances of the measurement points for the four cases. The distances are measured from the left end of the slab. Only four measurement points with different locations are considered in this part of the study.

As there are a total of 33 nodes in a complete mode shape, the remaining 29 points will be determined from the CS method and ANN model. To assess the effectiveness of the proposed method, the mode shapes obtained from both methods are then compared with the finite element mode shapes of damage in Case A, which are assumed to be the actual mode shapes. Figure 8 exhibits all the mode shape curves obtained from both the CS method and the ANN.

#### Table 7 Location of measurement points (distance from left in meters)

<table>
<thead>
<tr>
<th>Case</th>
<th>First point</th>
<th>Second point</th>
<th>Third point</th>
<th>Fourth point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0</td>
<td>2.2</td>
<td>4.2</td>
<td>6.4</td>
</tr>
<tr>
<td>Case 2</td>
<td>0</td>
<td>1.2</td>
<td>5.2</td>
<td>6.4</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.6</td>
<td>2.4</td>
<td>4.0</td>
<td>5.8</td>
</tr>
<tr>
<td>Case 4</td>
<td>0</td>
<td>2.4</td>
<td>5.2</td>
<td>6.4</td>
</tr>
</tbody>
</table>

The results show that the ANN is able to predict all three mode shapes accurately for all cases. Regardless of the location of the measurement points, the ANN model is still able to predict the unmeasured mode shape values accurately once it is well trained. This is in contradiction to the CS method, where in the first mode, although the CS method is able to obtain the correct shape in all four cases, the error is quite remarkable. For higher modes (2 and 3), the CS method is unable to provide satisfactory results for all cases. Moreover, for Cases 1, 2, and 4 with the same two measurement points at both ends, the CS interpolated mode shapes are unable to match the finite element mode shape. This clearly shows that the CS method is very dependent on the location of measurement points, while the ANN, once trained with sufficient data, is able to predict the mode shape curves correctly. This proves that the CS method is more sensitive to measurement locations than the ANN method. Therefore, by using the ANN approach, the use of a rigorous process to select the measurement points for damage detection can be avoided.

### 3.2 Damage Detection Using Two-Stage ANN

This section demonstrates the application of ANN2 for damage detection. The inputs to the ANN2 are a combination of measured mode shapes and frequency values and the predicted mode shape values at the unmeasured locations obtained from the ANN1. The outputs are the SRF of each segment of the slab. The sensitivity of ANN2 is compared to that of the conventional ANN model. The architecture of the conventional ANN model is as discussed in Section 2.1.

To maintain the consistency of the study, both the two-stage ANN and the conventional ANN models are trained using the same training data generated from the finite element model. Both the conventional ANN and the ANN1 models are trained using the first three modes of frequencies and mode shapes from 4, 6, 8, and 16 measurement points as the input parameters. The outputs of the conventional ANN models are the SRF of each segment of the slab, while for ANN1 they are the remaining unmeasured points of the slab. The locations of the sensors for the different cases of numbers of measurement points are as discussed previously. Similarly, 3000 training data are used in the training session. For the two-stage ANN, the outputs of ANN1 are fed into ANN2 together with the measured frequencies and mode shapes to predict the SRF of each element of the slab.

For comparison, the simulated damage scenario of Case B using 16 measurement points is used in this part to investigate the feasibility of the proposed two-stage ANN approach. The comparisons of the damage prediction are measured using the MSE. The MSE values are calculated based on the actual damage.
Figure 9 depicts the summary of the comparison of MSE values for all of the ANN models in Case B. The results show that the proposed method provides lower MSE values than the conventional ANN models with a lower number of measurement points (4 and 6). This indicates that ANN1 is able to provide additional information to ANN2 to improve the damage prediction performance when a small number of measurements is used. When the number of measurement points is low, the input fed into the conventional ANN is very limited. Hence, the inadequate input causes the generalisation results to be poor. However, as the number of measurement points.

![Mode shape curves influenced by different measurement points](image1)

**Figure 8** Mode shape curves influenced by different measurement points

![MSE comparison between single-stage and two-stage ANN (Case B)](image2)

**Figure 9** MSE comparison between single-stage and two-stage ANN (Case B)

Although the effect of ANN prediction propagation errors from the earlier ANN1 model has reduced the efficiency of the two-stage ANN model, in practice it is not always economical to measure the mode shape at a high number of measurement points.
points. Figure 10 shows the damage prediction results for Cases A and B using 16 measurement points in the two-stage ANN. It is observed that ANN2 is able to locate the damage location correctly; however, all the SRF values are slightly underestimated. There is also some minor negative and positive false detection on the left and right of the damaged segments in both damage cases. This is attributed to the propagation errors mentioned earlier. The negative values of SRF in the Figure 10 are assumed to be no damage. In comparison with the conventional ANN model, the proposed method is able to enhance the damage detection result when a limited number of measurement points are available; thus, by using the proposed approach, the number of measurement points can be kept small. The findings of this study could be used to overcome the issue of using a limited number of measurement points for damage detection.

![Actual Damage vs Predicted Damage](image)

**Figure 10** Generalisation of 16P models (two-stage ANN)

### 4.0 CONCLUSION

The study has demonstrated an approach that applies ANNs to predict unmeasured mode shape values for damage detection. Two simulated damage cases are created to test the feasibility of the proposed method. In the first-stage ANN, a comparison with the CS method showed that application of an ANN predicted the mode shape values at the unmeasured points better. The proposed two-stage ANN is able to accurately predict the mode shape curves and to identify the damage locations. Furthermore, the proposed approach can also be used to overcome the problem of trial and error in selecting the measurement points for damage detection. However, the propagation errors induced by ANN1 outputs have reduced the efficiency for damage detection. Hence, an improvement to the proposed approach by considering the propagation errors will be a focus of future studies.

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